Homework exercises Type Theory, 28/2/2007

1. Since the embedding of combinatory logic into λ-calculus is surjective on equivalence classes, there must be fixed point combinators M in combinatory logic, characterized by $Mx \twoheadrightarrow_w x(Mx)$. Give such a fixed point combinator. Hint: you can use $\mathbf{I}, \mathbf{B}, \mathbf{C}, \ldots$ as shorthands and define other subterms separately.

2. (a) Proof lemma 5.2.3: if $\Gamma, x : \rho \vdash_{\mathcal{C}} F : \tau$, then $\Gamma \vdash_{\mathcal{C}} \lambda^* x.F : \rho \to \tau$

   (b) Proof proposition 5.3.4: the deduction theorem for Hilbert-style proofs. Hint: Hilbert-style proofs correspond to combinators, and the deduction theorem corresponds to the λ^* abstaction operator under the Curry Howard isomorphism.

3. Find a tautology of the implicational fragment of intuïtionistic propositional logic, that is not provable in relevance logic and **BCK**-logic.