Proximity of Persistence Modules and their Diagrams

Topological persistence has proven to be a key concept for the study of real-valued functions defined over topological spaces. Its validity relies on the fundamental property that the persistence diagrams of nearby functions are close. However, existing stability results are restricted to the case of continuous functions defined over triangulable spaces.

In this paper, we present new stability results that do not suffer from the above restrictions. Furthermore, by working at an algebraic level directly, we make it possible to compare the persistence diagrams of functions defined over different spaces, thus enabling a variety of new applications of the concept of persistence. Along the way, we extend the definition of persistence diagram to a larger setting, introduce the notions of discretization of a persistence module and associated pixelization map, define a proximity measure between persistence modules, and show how to interpolate between persistence modules, thereby lending a more analytic character to this otherwise algebraic setting. We believe these new theoretical concepts and tools shed new light on the theory of persistence, in addition to simplifying proofs and enabling new applications.