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Lower Bounds for Weak epsilon-Nets and Stair-Convexity

A subset $N$ of $\mathbb{R}^d$ is called a "weak epsilon-net" (with respect to convex sets) for a finite point set $X$ in $\mathbb{R}^d$ if $N$ intersects every convex set that contains at least $\varepsilon |X|$ points of $X$. For every fixed $d \geq 2$ and every $r \geq 1$ we construct subsets $X$ of $\mathbb{R}^d$ for which every weak $1/r$-net has at least $\Omega(r \log^{d-1} r)$ points; this is the first superlinear lower bound for weak epsilon-nets in a fixed dimension.