We consider the problem of computing minimum-sized geometric hitting sets in which, given a set of geometric objects and a set of points, the goal is to compute the smallest subset of points which hit all geometric objects. The problem is known to be strongly NP-hard even for simple geometric objects like unit disks in the plane. Therefore, unless P = NP, it is not possible to get Fully Polynomial Time Approximation Algorithms (FPTAS) for such problems. We give the first PTAS for this problem when the geometric objects are halfspaces in $\mathbb{R}^3$ and when they are $r$-admissible regions in the plane (this includes pseudodiscs since they are 2-admissible). When there are $m$ objects and $n$ points, the algorithm computes a $(1 + \varepsilon)$-approximation to the minimum hitting set in time $O(mn^{O(\varepsilon^{-2})})$. Quite surprisingly, our algorithm is a very simple local search algorithm which iterates over local improvements.