Two geometric medians (≥ 3 points)



The optimal bisector (defined by the two geometric medians) partitions the points into two sets A and B. **Claim** There exists a line through two input points, that separates A and B, where the two points are *either* both in A or both in B.

Proof. Consider the cases...

Case 1: Parallel shift bisector away from side with most points, until reaching point p_1 , and turn line until it touches p_2 or p_3 on the same side. Line p_1p_2 or p_1p_3 is a valid bisector.



Case 2: Parallel shift bisector away from side with most points, until reaching point p_1 , and turn line until it touches the first points p_2 and p_3 on other side. Assume wlog. p_2 is furtherst away from the true bisector. Turn line through p_2 until it touches the first point p_4 (possibly $p_4 = p_3$). The line p_2p_4 is a valid bisector.