## Two geometric medians ( $\geq 3$ points)



The optimal bisector (defined by the two geometric medians) partitions the points into two sets $A$ and $B$. Claim There exists a line through two input points, that separates $A$ and $B$, where the two points are either both in A or both in B .
Proof. Consider the cases...


Case 1: Parallel shift bisector away from side with most points, until reaching point $p_{1}$, and turn line until it touches $p_{2}$ or $p_{3}$ on the same side. Line $p_{1} p_{2}$ or $p_{1} p_{3}$ is a valid bisector.


Case 2: Parallel shift bisector away from side with most points, until reaching point $p_{1}$, and turn line until it touches the first points $p_{2}$ and $p_{3}$ on other side. Assume wlog. $p_{2}$ is furtherst away from the true bisector. Turn line through $p_{2}$ until it touches the first point $p_{4}$ (possibly $\left.p_{4}=p_{3}\right)$. The line $p_{2} p_{4}$ is a valid bisector.

