Grundlæggende Algoritmer og Datastrukturer

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WORST-CASE DATA STRUCTURES FOR THE PRIORITY QUEUE WITH ATTRITION

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We describe three data structures for the priority queue with attrition (or PQA) that perform each PQA operation in O(1) worst-case time. Previous implementations of the PQA required O(1) amortized time per operation.

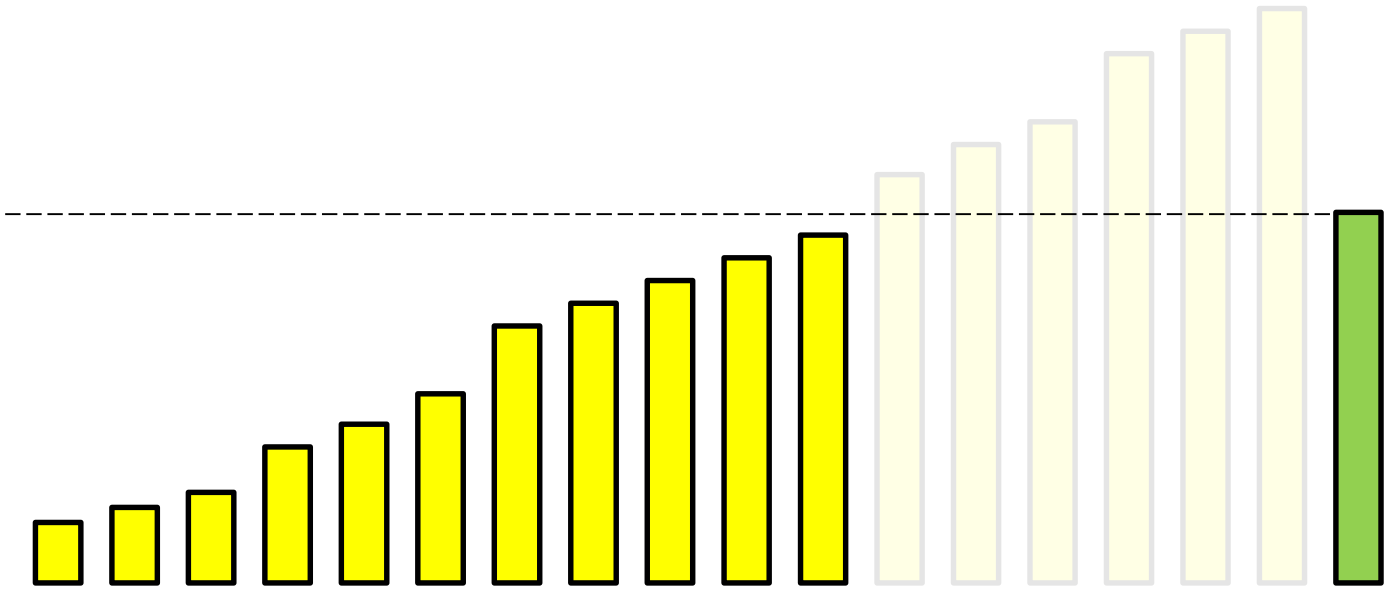
Keywords: Priority queue with attrition, worst-case

Operationer

Create $S := \emptyset$

Insert(*x*) $S := \{ x \} \cup \{ y \in S \mid y < x \}$

Deletemin $m := \min(S);\ S := S \setminus \{ m \};$ return m



Insert(4) i {1,3,5,6,9} ?

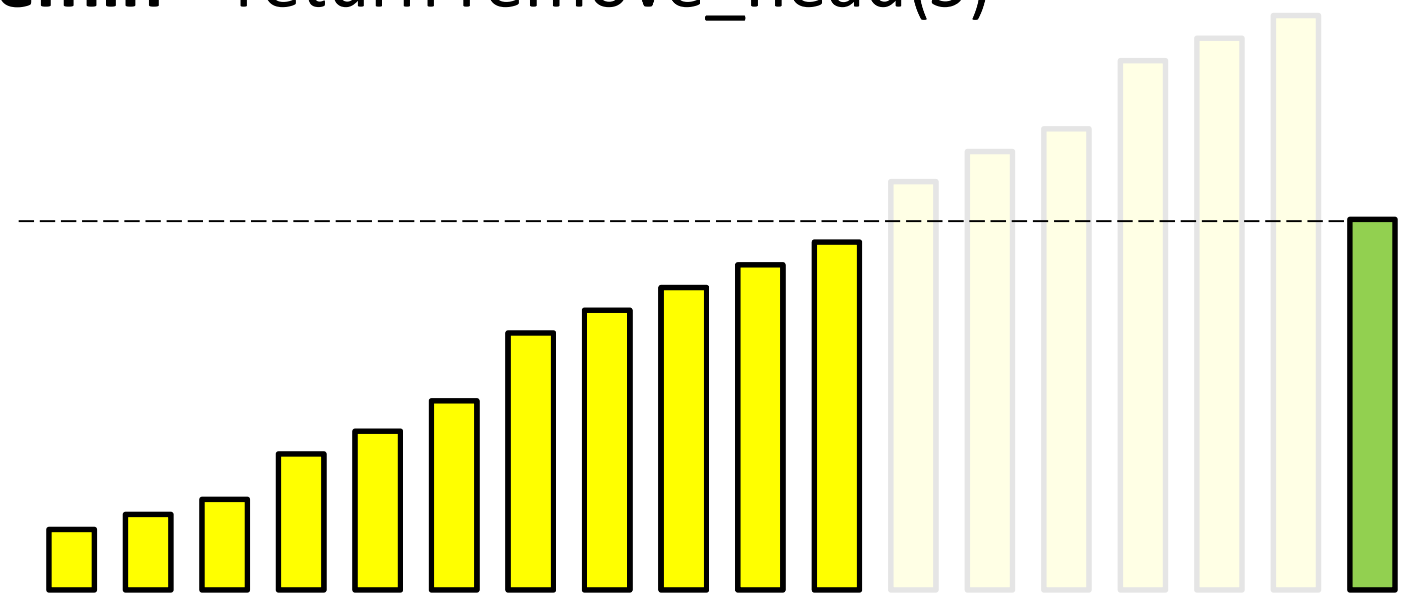
a) {4,5,6,9}
b) {1,3,4,5,6,9}
c) {1,3,4}
d) ved ikke

Løsning: Sorteret Liste

Create $S := ()$

Insert(*x*) while ($|S|>0$ and tail(S)$\geq x$) remove_tail(S)
insert_tail(x)

Deletemin return remove_head(S)



Løsning: Sorteret Liste

Create S := ()

Insert(*x*) while (|S|>0 and tail(S)≥x) remove_tail(S)
insert_tail(x)

Deletemin return remove_head(S)

Sætning
Create, Insert og DeleteMin tager **amortiseret** O(1) tid

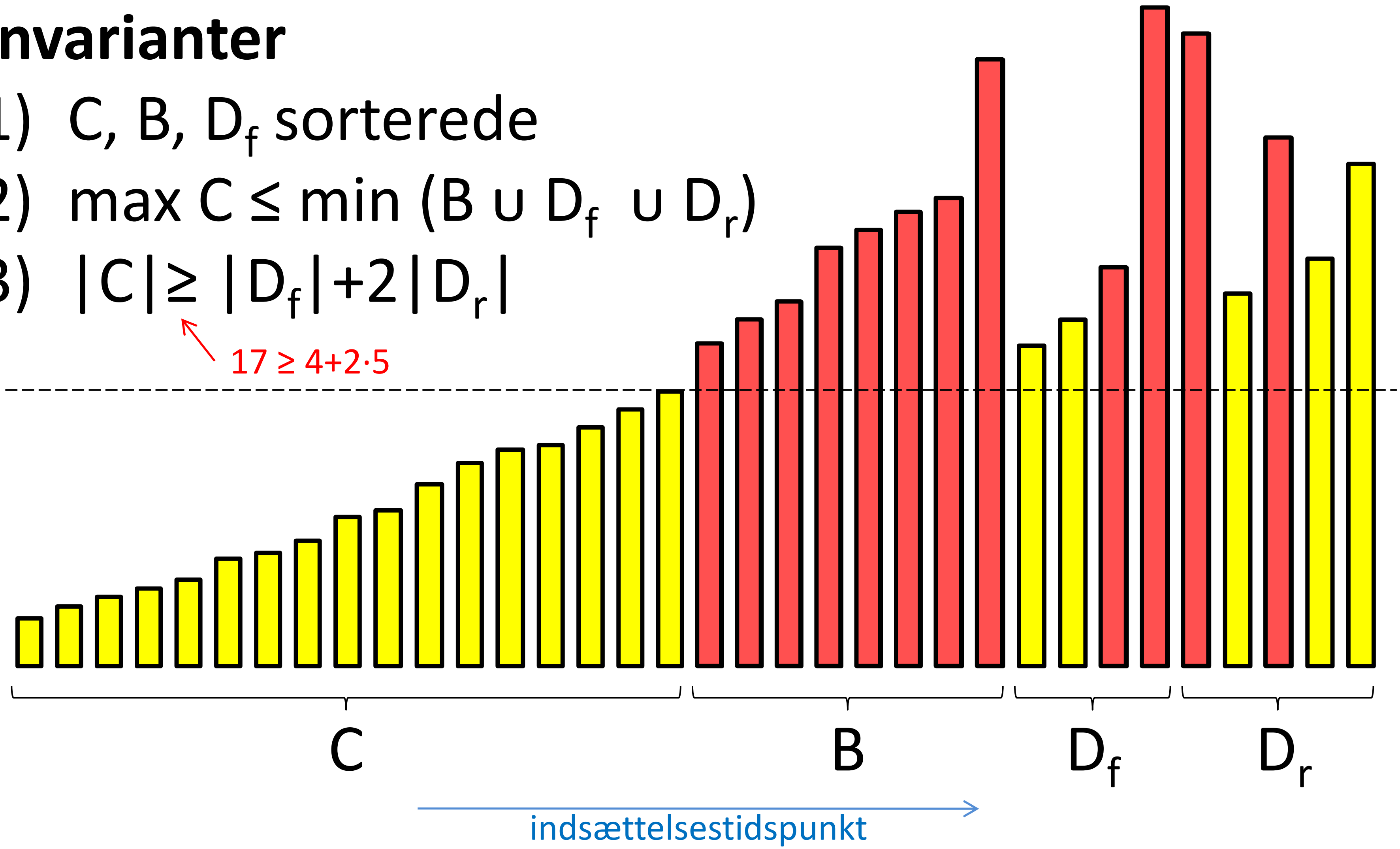
Bevis: $\Phi(S) = |S|$. □

Worst-Case O(1)

Løsning: 4 Lister

Invarianter

1) C, B, D_f sorterede
2) max C ≤ min (B ∪ D_f ∪ D_r)
3) $|C| \geq |D_f| + 2|D_r|$



Hvilken mængde repræsenteres ved:

(1,2,3,4,5,6)(7,10,14)(8,11)(9,8)

C B D_f D_r

a) {1,2,3,4,5,6,7,8,8,9,10,11,14}
b) {1,2,3,4,5,6,7,8,8,9}
c) {1,2,3,4,5,6,7,8,9}
d) {1,2,3,4,5,6,7,8}
e) {1,2,3,4,5,6,7,8,8}
f) ved ikke

CREATEPQA ≡

C, B, D_f, D_r := ( ), ( ), ( ), ( )

INSERT(x) ≡

if $C \neq ( )$ cand first(C) $\geqslant x$ then

{Delete all existing items; add x to C}

1 C, B, D_f, D_r := (x), ( ), ( ), ( )

else if $C \neq ( )$ cand last(C) $\geqslant x$ then

{Empty B, D_f, and D_r; push back rest(C) into B; add x to D_f}

2 C, B, D_f, D_r := (first(C)), rest(C), (x), ( )

3 else $D_r := D_r \| (x)$; BIAS; BIAS

DELETEMIN ≡

BIAS;

return DELETEFIRST(C)

{BIAS ensures that $C \neq ( )$, unless the PQA is empty}

Invarianter

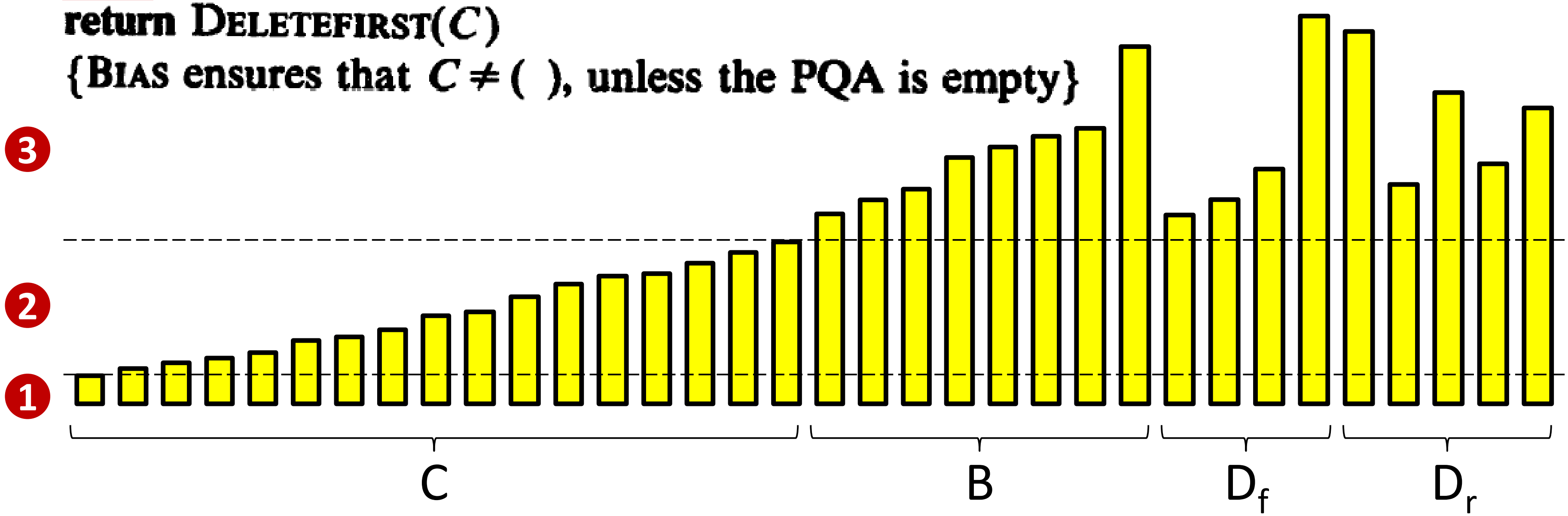
1) C, B, D_f sorterede

2) $\max C \leq \min (B \cup D_f \cup D_r)$

3) $|C| \geq |D_f| + 2|D_r|$

≥ "+1"

BIAS

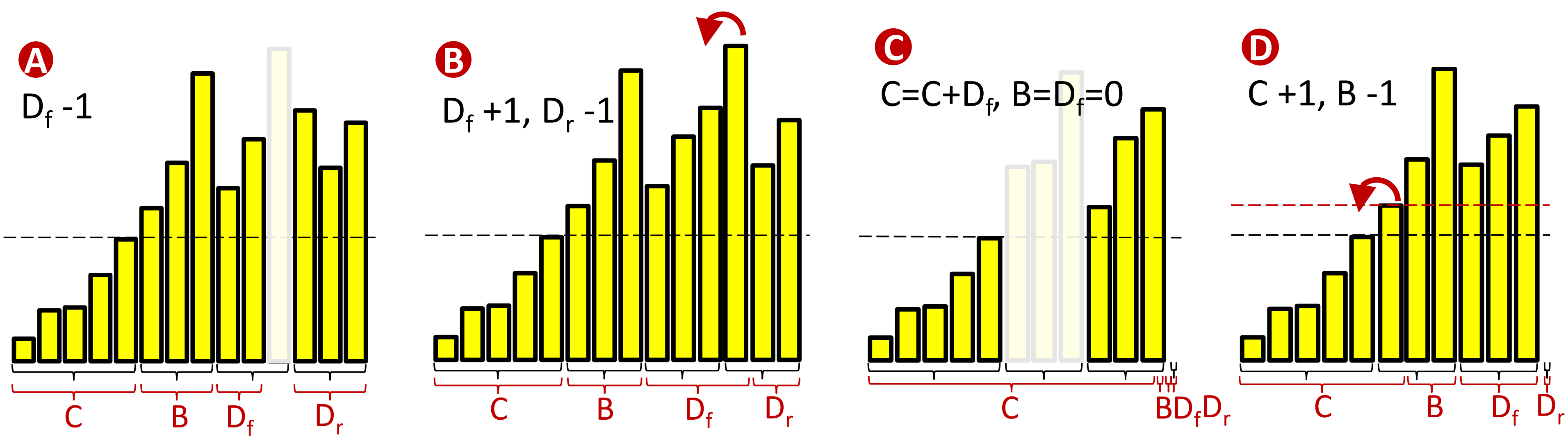


Invarianter

1) C, B, D_f sorterede
2) max C ≤ min (B ∪ D_f ∪ D_r)
3) $|C| \geq |D_f| + 2|D_r|$

≥ "+1" BIAS

BIAS ≡
  if $D_r \neq (\ )$ **then**
    {Clean-up step}
    if $D_f \neq (\ )$ **cand** last(D_f) ≥ first(D_r) **then**
A   DELETELAST(D_f) {decrease $|D_f|$}
B   **else** PASS(D_r, D_f) {decrease $|D_r|$; increase $|D_f|$}
  else if $D_f \neq (\ )$ **cand** ($B = (\ )$ **cor** first(B) ≥ first(D_f)) **then**
C $D_f, B, C := (\ ), (\ ), C \,\|\, D_f$ {decrease $|D_f|$; increase $|C|$}
D **else if** $B \neq (\ )$ **then** PASS(B, C) {increase $|C|$}
  {else $B = D_f = D_r = (\ )$}



Resultatet af **insert(7)** på:

(1,2,3,4,5,6)(7,10,14)(8,11)(9,8)

C B D_f D_r

a) (1,2,3,4,5,6)(7,10,14)(8,11)(9,8,7)

b) (1,2,3,4,5,6)(7,10,14)(8,9)(8,7)

c) (1,2,3,4,5,6)(7,10,14)(8)(9,8,7)

d) (1,2,3,4,5,6,7)(10,14)(8)(9,8,7)

e) ved ikke

Sætning

Create, Insert og DeleteMin tager **worst-case** O(1) tid

Invarianter

1) C, B, D_f sorterede
2) max C ≤ min ($B \cup D_f \cup D_r$)
3) $|C| \geq |D_f| + 2|D_r|$