Algoritmer og Datastrukturer 1

Gerth Stølting Brodal

Elementære Datastrukturer [CLRS, kapitel 10]

AARHUS UNIVERSITET

[CLRS, Del 3] : Datastrukturer

Opretholde en struktur for en **dynamisk** mængde data

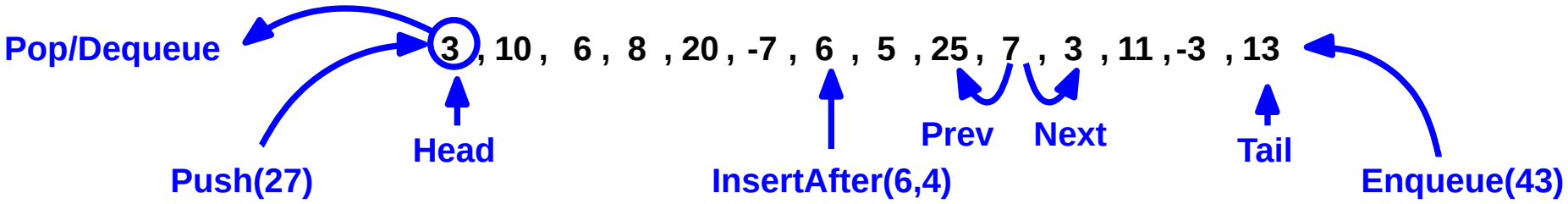
Abstrakte Datastrukturer for Mængder

-Min-prioritetskø
-Max-prioritetskø
- Ordbog

Forespørgsel	Minimum(S)	pointer til element	●		
	Maximum(S)	pointer til element		●	
	Search(S, x)	pointer til element			●
	Member(S, x)	TRUE eller FALSE			
	Successor(S, x)	pointer til element			
	Predecessor(S, x)	pointer til element			
Opdateringer	Insert(S, x)	pointer til element	●	●	●
	Delete(S, x)	-			●
	DeleteMin(S)	element	●		
	DeleteMax(S)	element		●	
	Join(S_1, S_2)	mængde S			
	Split(S, x)	mængder S_1 og S_2			

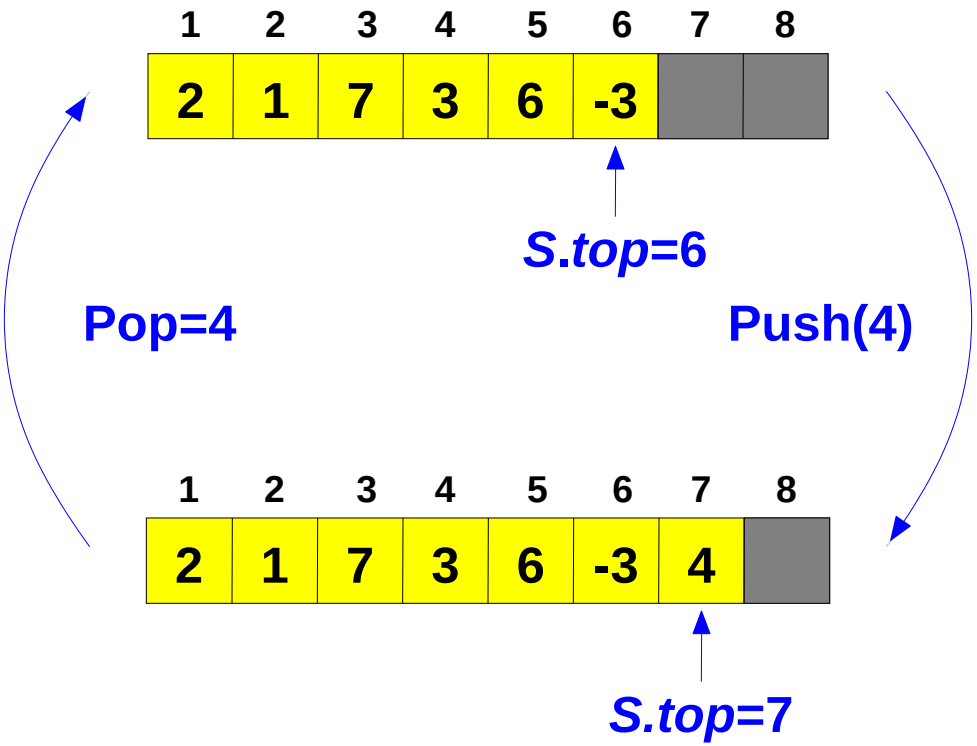
Abstrakte Datastrukturer for Lister

			-Stak	-Kø
Forespørgsel	Empty(S)	TRUE eller FALSE	●	●
	Head(S), Tail(S)	pointer til element		
	Next(S, x), Prev(S, x)	pointer til element		
	Search(S, x)	pointer til element		
Opdateringer	Push(S, x)	-	●	
	Pop/Dequeue(S)	element	●	●
	Enqueue(S, x)	-		●
	Delete(S, x)	Element		
	InsertAfter(S, x, y)	pointer til element		



Stak

Stak : Array Implementation



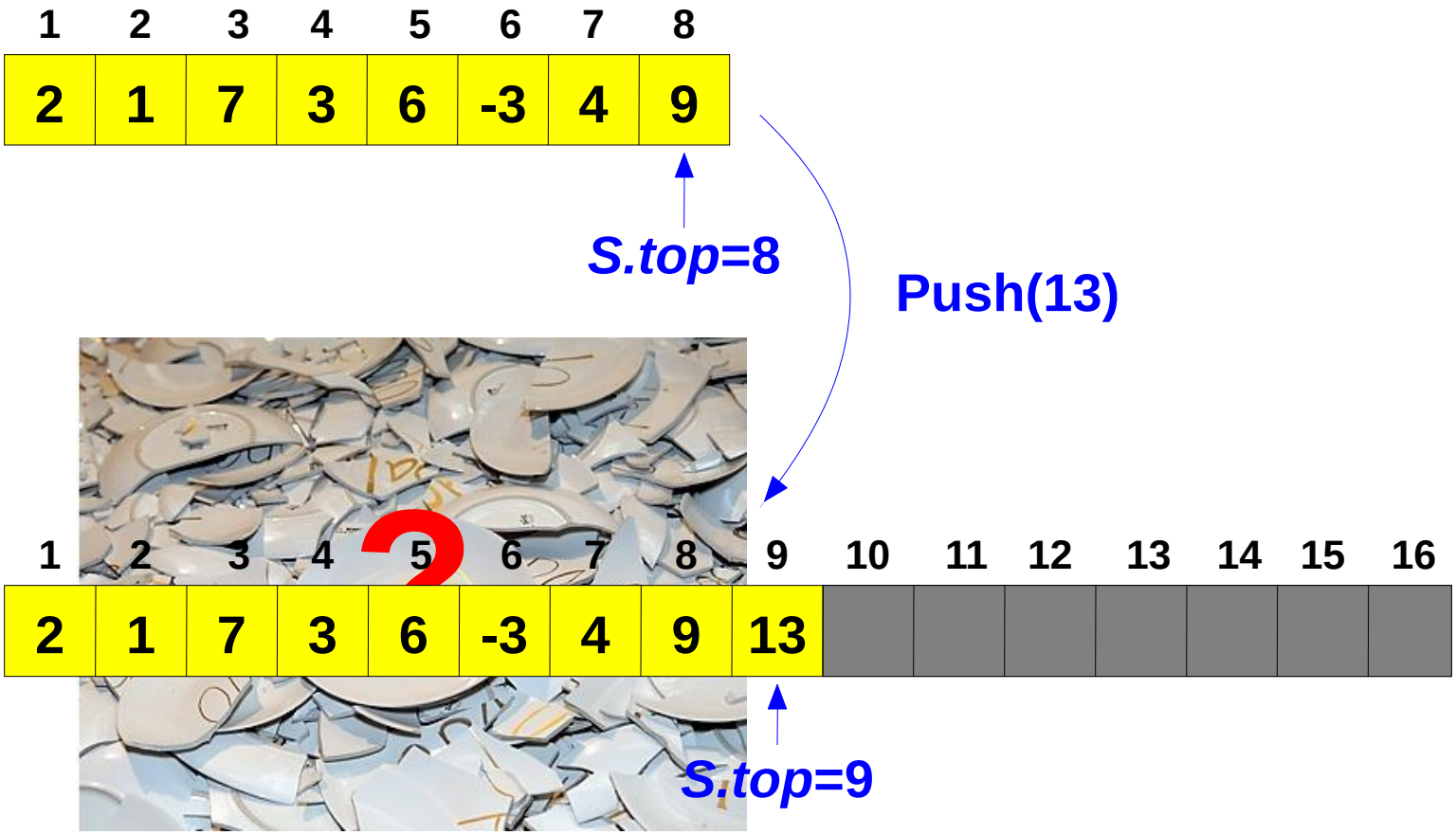

```
STACK-EMPTY(S)
1  if S.top == 0
2       return TRUE
3  else return FALSE
```

```
PUSH(S, x)
1  S.top = S.top + 1
2  S[S.top] = x
```

```
POP(S)
1  if STACK-EMPTY(S)
2       error "underflow"
3  else S.top = S.top - 1
4       return S[S.top + 1]
```

Stack-Empty, Push, Pop : O(1) tid

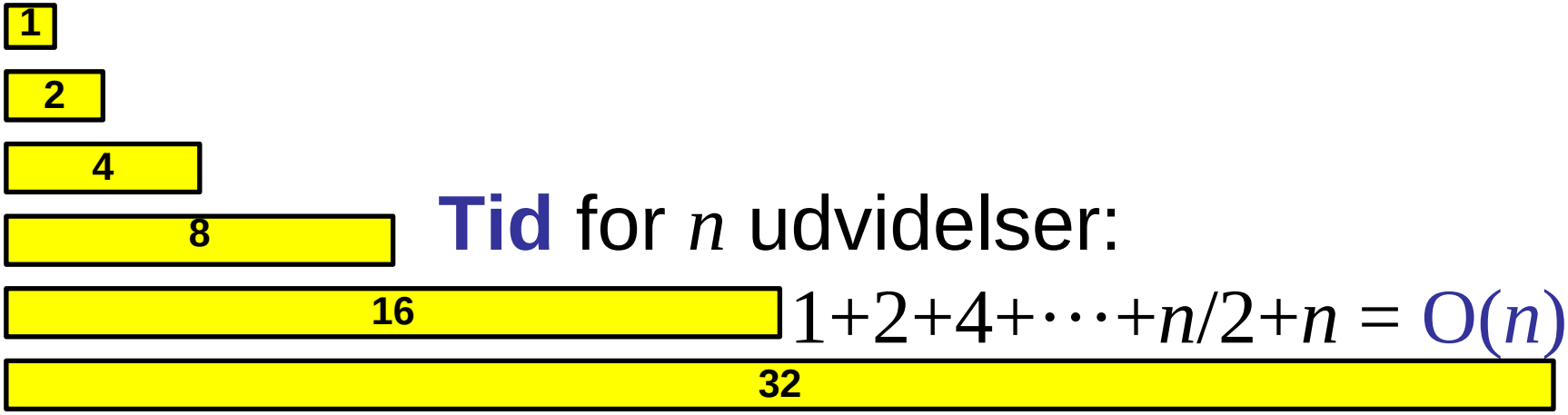
Stak : Overløb



Array fordobling : $O(n)$ tid

Array Fordobling

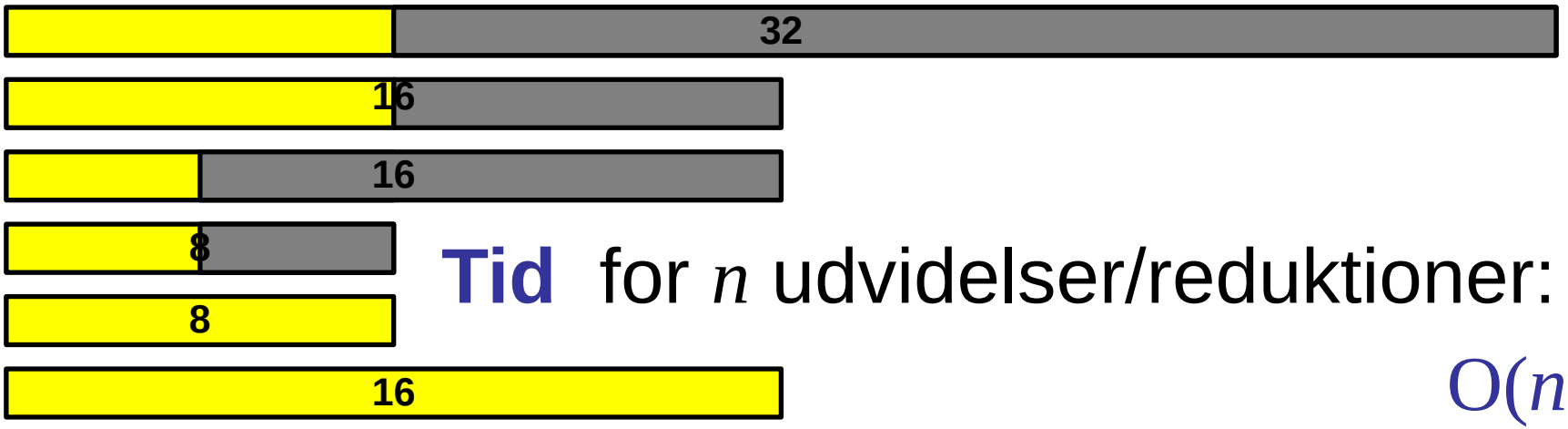
Fordoble arrayet når det er fuld



Tid for n udvidelser:

$1+2+4+\cdots+n/2+n = O(n)$

Halver arrayet når det er <1/4 fyldt



Tid for n udvidelser/reduktioner:

$O(n)$

Array Fordobling + Halvering
– en generel teknik

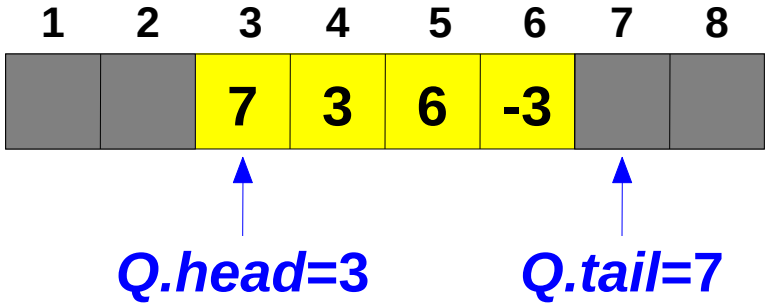
Tid for n udvidelser/reduktioner er $O(n)$

Plads $\leq$ 4 $\cdot$ aktuelle antal elementer

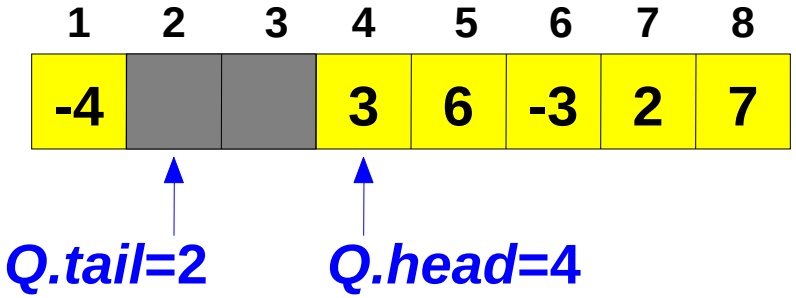
Array implementation af Stak:
n push og pop operationer tager $O(n)$ tid

Kø

Kø : Array Implementation



Enqueue(2)
Enqueue(7)
Engueue(-4)
Dequeue = 7



ENQUEUE(Q, x)

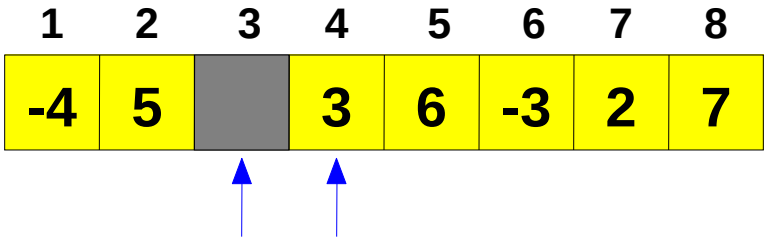
```
1  Q[Q.tail] = x
2  if Q.tail == Q.length
3        Q.tail = 1
4  else Q.tail = Q.tail + 1
```

DEQUEUE(Q)

```
1  x = Q[Q.head]
2  if Q.head == Q.length
3        Q.head = 1
4  else Q.head = Q.head + 1
5  return x
```

Enqueue, dequeue : $O(1)$ tid

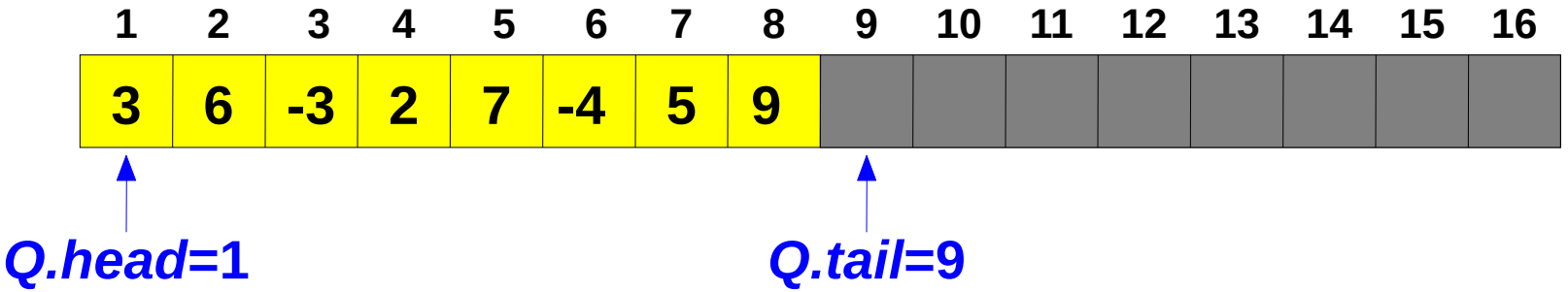
Kø : Array Implementation



Empty : *Q.tail*=*Q.head* ?

Overløb : array fordobling/ halvering

Enqueue(9)



Array implementation af Kø:
n enqueue og dequeue operationer tager $O(n)$ tid

Arrays (med Fordobling/Halvering)

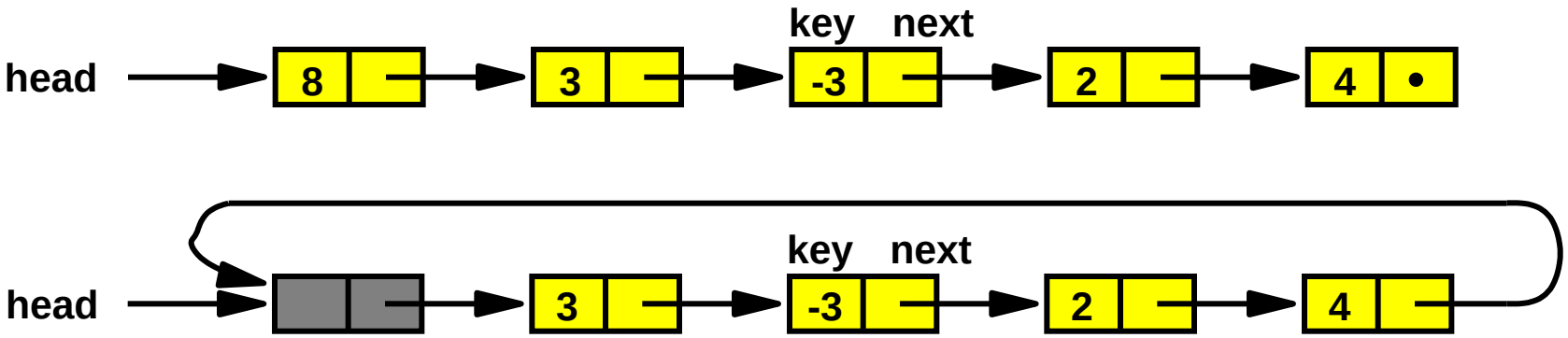
Stak	**Push(S, x)**	O(1)*
	Pop(S)	O(1)*
Kø	**Enqueue(S, x)**	O(1)*
	Dequeue(S)	O(1)*

*** Worst-case uden fordobling/halvering**
Amortiseret ([CLRS, Kap. 17]) med fordobling/halvering

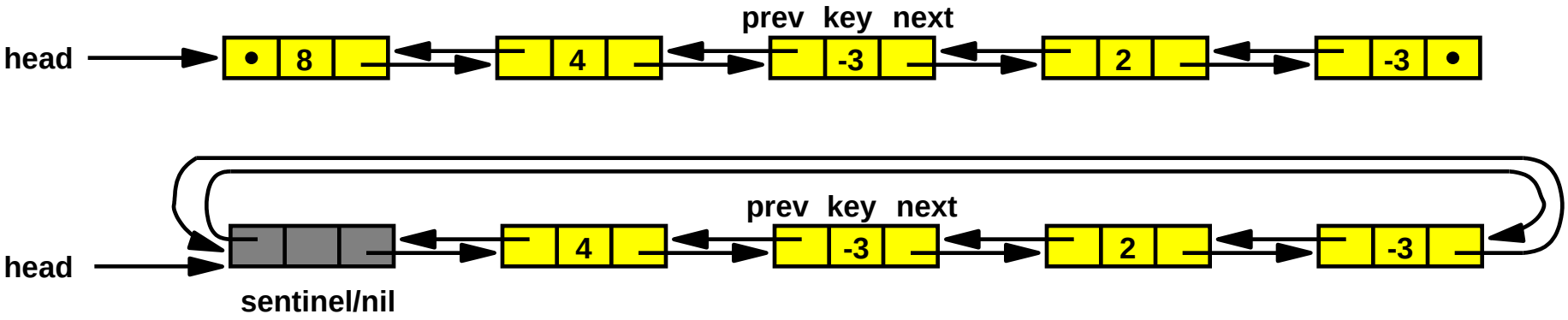
Kædede lister

Kædede Lister

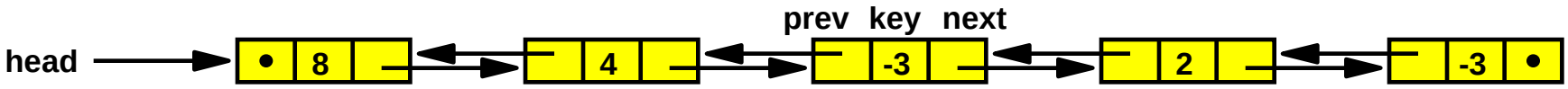
Enkelt kædede (ikke-cyklisk og cyklisk)



Dobbelt kædede (ikke-cyklisk og cyklisk)



Dobbelt Kædede Lister



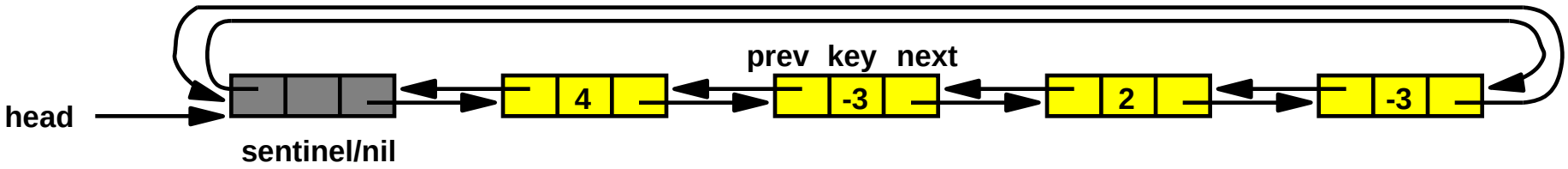

```
LIST-SEARCH(L, k)
1  x = L.head
2  while x ≠ NIL and x.key ≠ k
3      x = x.next
4  return x
```

```
LIST-INSERT(L, x)
1  x.next = L.head
2  if L.head ≠ NIL
3      L.head.prev = x
4  L.head = x
5  x.prev = NIL
```

```
LIST-DELETE(L, x)
1  if x.prev ≠ NIL
2      x.prev.next = x.next
3  else L.head = x.next
4  if x.next ≠ NIL
5      x.next.prev = x.prev
```

List-Search	O(n)
List-Insert	O(1)
List-Delete	O(1)

Dobbelt Kædede Cykliske Lister



LIST-SEARCH′(L, k)

```
1  x = L.nil.next
2  while x ≠ L.nil and x.key ≠ k
3        x = x.next
4  return x
```

LIST-INSERT′(L, x)

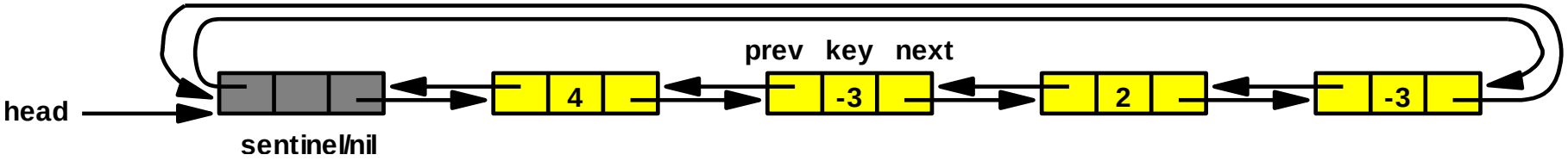
```
1  x.next = L.nil.next
2  L.nil.next.prev = x
3  L.nil.next = x
4  x.prev = L.nil
```

LIST-DELETE′(L, x)

```
1  x.prev.next = x.next
2  x.next.prev = x.prev
```

List-Search'	$O(n)$
List-Insert'	$O(1)$
List-Delete'	$O(1)$

Dobbelt Kædede Cykliske Lister



Stak	**Push(S, x)**	O(1)
	Pop(S)	O(1)
Kø	**Enqueue(S, x)**	O(1)
	Dequeue(S)	O(1)

Dancing Links

Donald E. Knuth, Stanford University

My purpose is to discuss an extremely simple technique that deserves to be better known. Suppose x points to an element of a doubly linked list; let $L[x]$ and $R[x]$ point to the predecessor and successor of that element. Then the operations

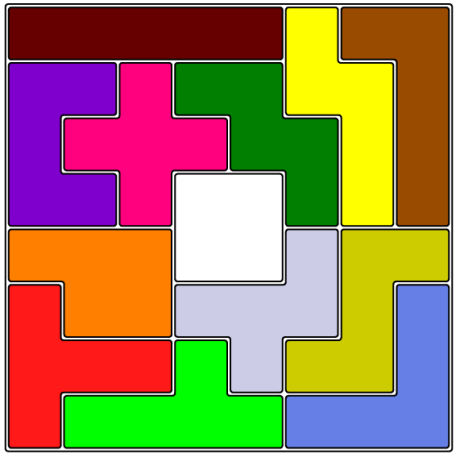
$$L\big[R[x]\big] \leftarrow L[x], \qquad R\big[L[x]\big] \leftarrow R[x] \tag{1}$$

remove x from the list; every programmer knows this. But comparatively few programmers have realized that the subsequent operations

$$L\big[R[x]\big] \leftarrow x, \qquad R\big[L[x]\big] \leftarrow x \tag{2}$$

will put x back into the list again.

Donald E. Knuth (1938-)



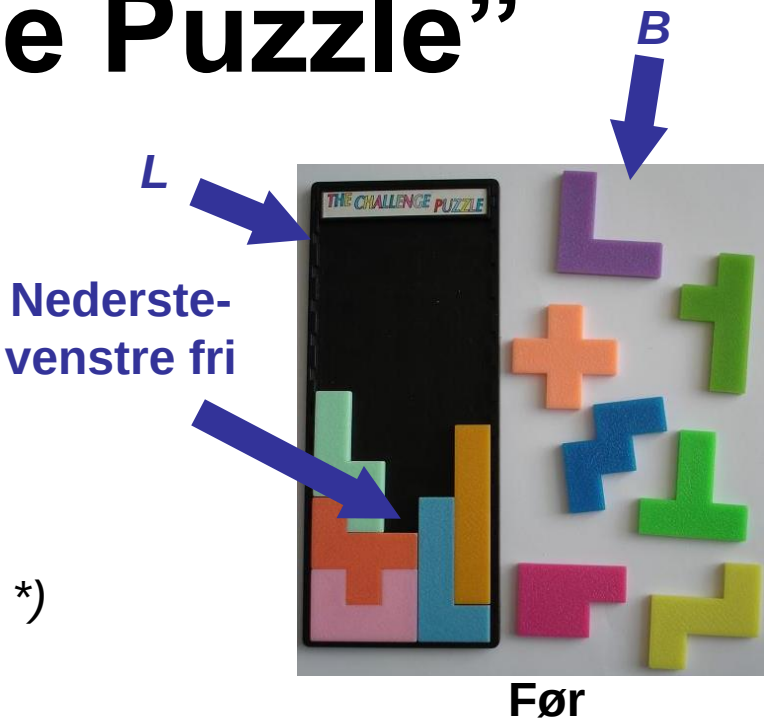
”The Challenge Puzzle”

?

”The Challenge Puzzle”

L := Tomt bræt
B := Alle brikker
Solve(*L*,*B*)

procedure Solve(Delløsning *L*, Brikker *B*)
  for alle *b* i *B*
    for alle orienteringer af *b* *(* max 8 forskellige *)*
      if *b* kan placeres i nederste venstre fri **then**
        fjern *b* fra *B*
        indsæt *b* i *L*
        if |*B*|=0 **then**
          rapporter *L* er en løsning
        else
          Solve(*L*,*B*)
        fi
        slet *b* fra *L*
        genindsæt *b* i *B*
      fi



Før

Efter

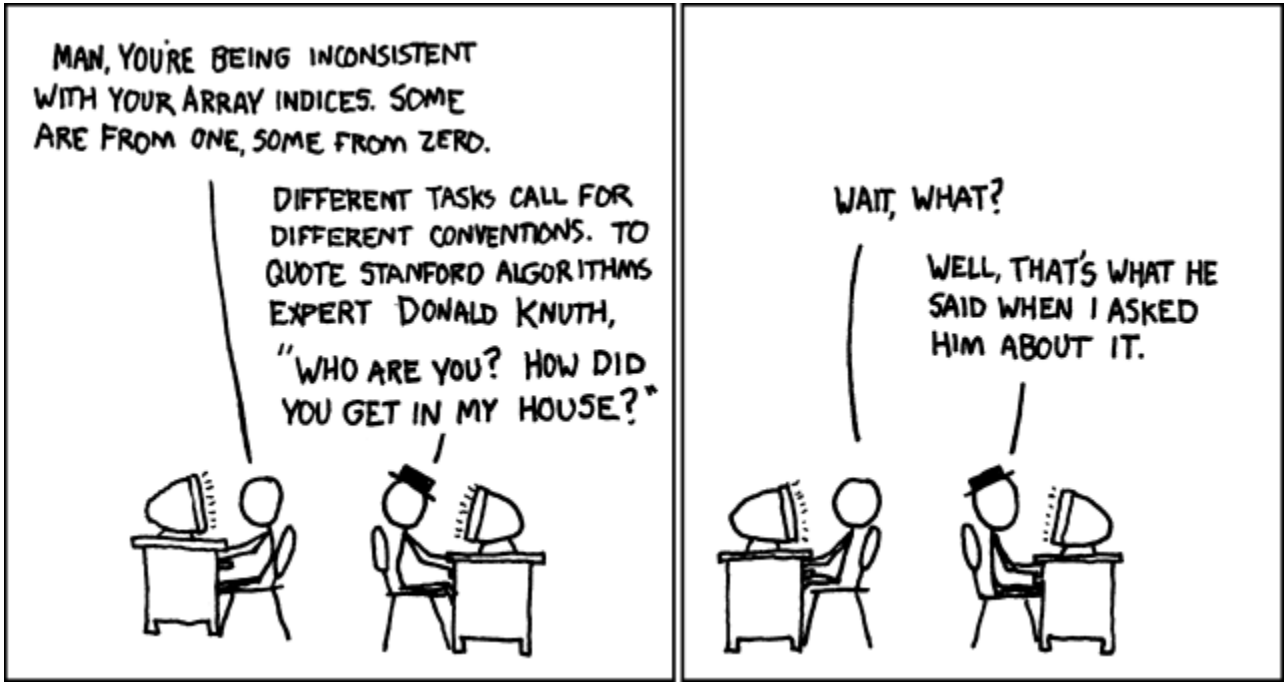
”The Challenge Puzzle”

4.040 løsninger

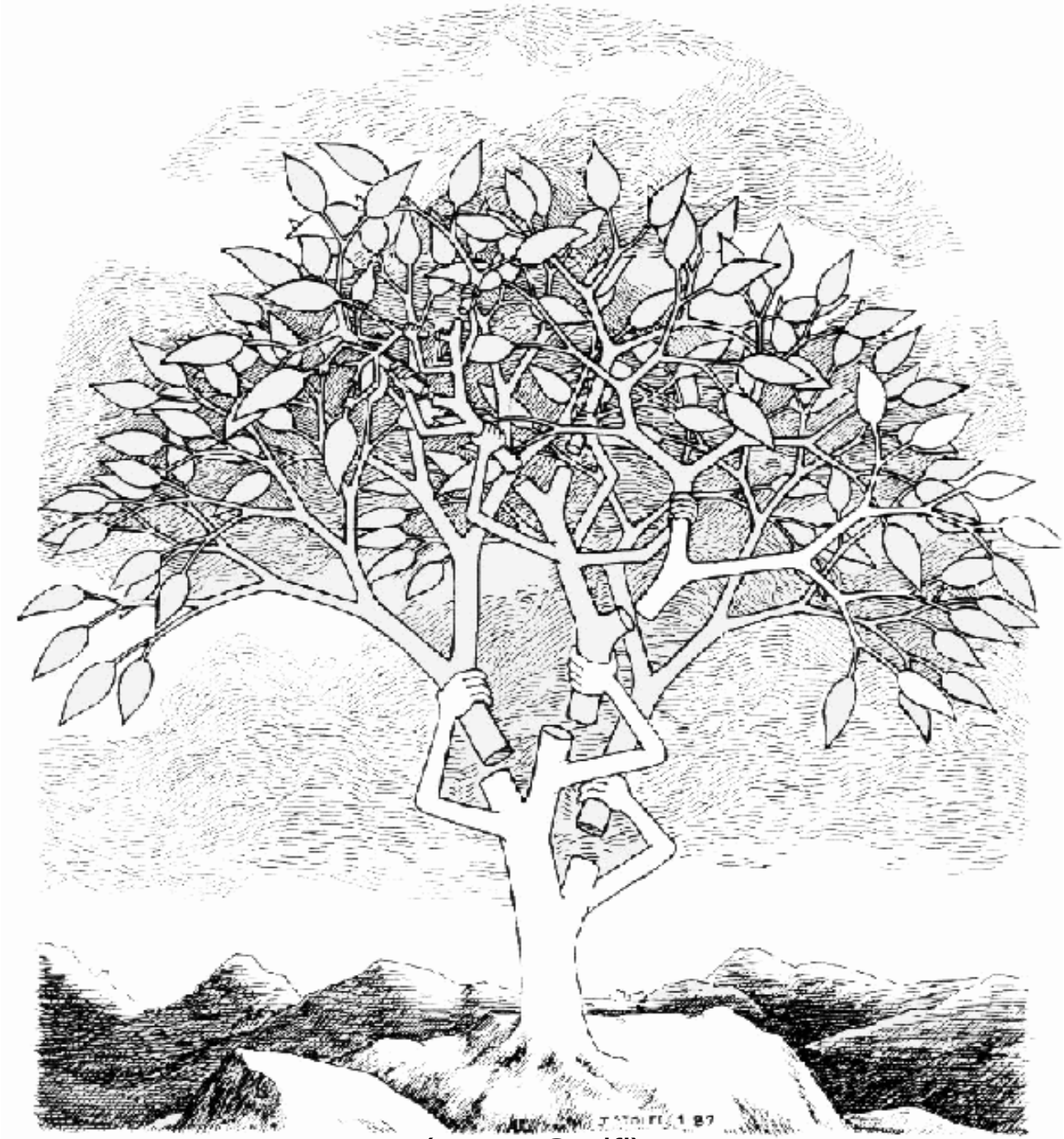
Solve placerer

8.387.259 brikker

Donald Knuth

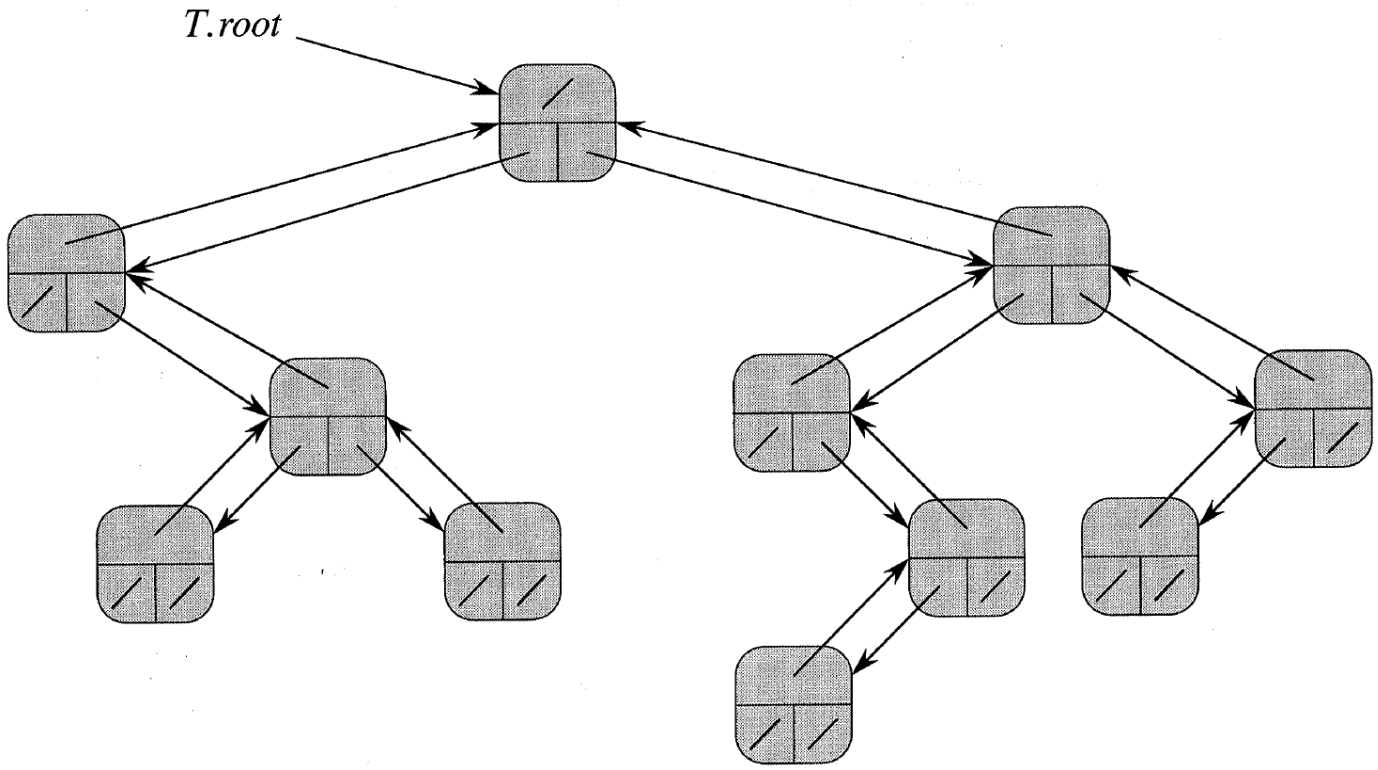


xkcd.com/163



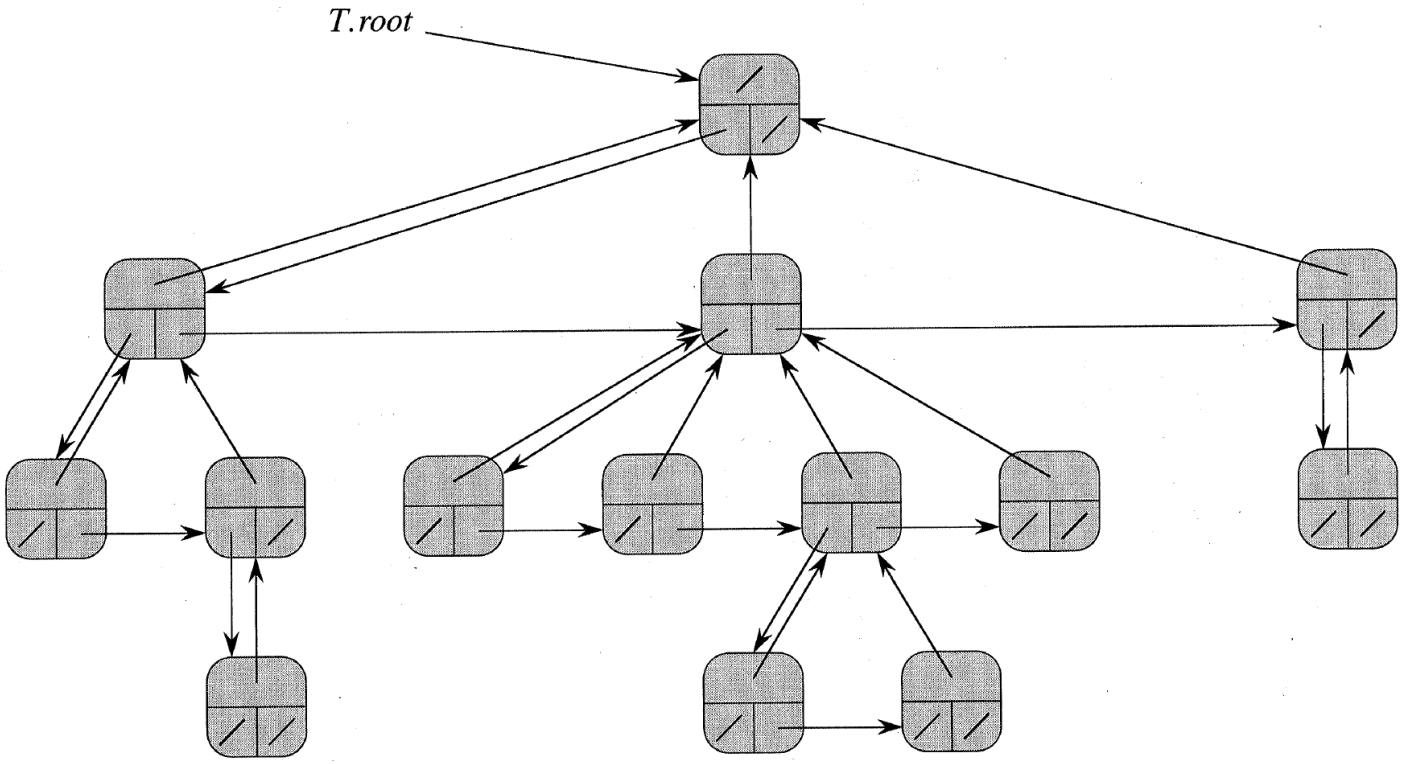
(Jorge Stolfi)

Binær Træ Repræsentation



Felter: **Left**, **right**, **parent**

Træ Repræsentation



Felter: **Left**, **right sibling**, **parent**