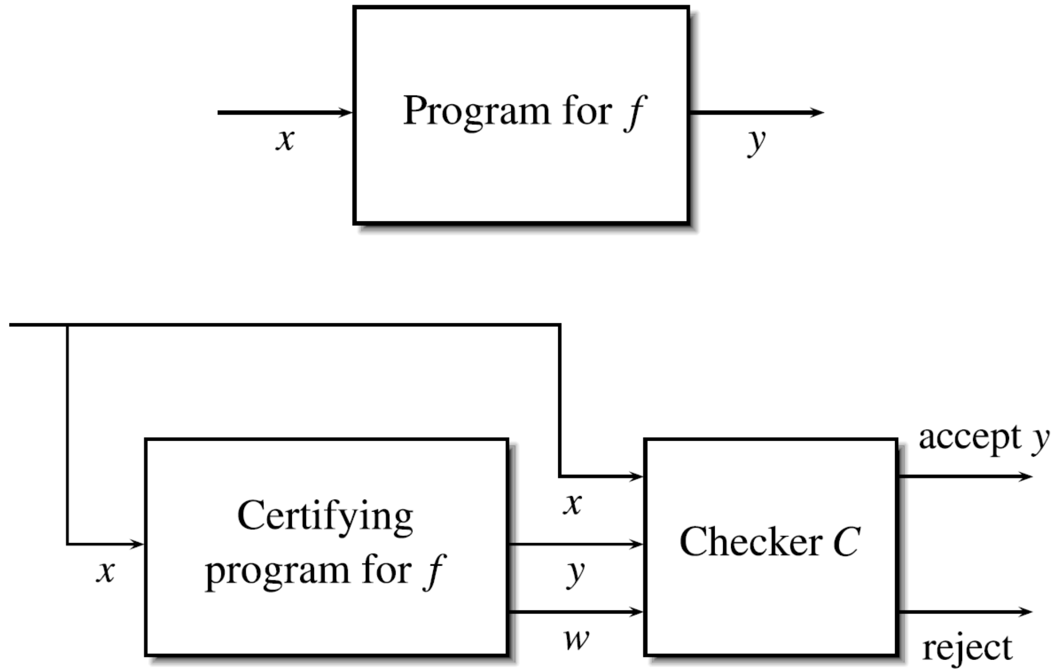
Certifying Algorithms

[MNS11] R.M. McConnell, K. Mehlhorn, S. Näher, P. Schweitzer. Certifying algorithms. Computer Science Review, 5(2), 119-161, 2011.

Correctnesss of algorithms ?

- Formal proof of algorithm correctness
  - only simple problems ?
  - implementation $\neq$ algorithm
- Compare output of two algorithms
  - one algorithm often simple and slow (only small input)
- Assertions / exceptions
- Unit testing
  - systematic testing, random input

Certifying Algorithm



- Algorithms output proof *w* of **correctnes** or **illegal input**
- **Strongly certifying** $\Rightarrow$ halts on all input; identifies illegal input
- **Certifying** $\Rightarrow$ halts on all input; illegal input *or* correct output
- **Weakly certifying** $\Rightarrow$ halts on valid input; if halts, correct out
- **Motivation**: Ensure correctnes of algorithms in the *Library of Efficient Data Types and Algorithms*

Sorting ?

- **Input**: An array of elements
- **Output**: Input elements in sorted order

- Checker:
  - Verify output sorted
  - Verify output = input elements

Greatest Common Divisor - GCD

- **Input**: Positive integers *a* and *b*
- **Output**: g = gcd(*a*, *b*)
- Certificate:
  - Integers *x*, *y*: where $g = ax + by$
- Checker:
  - Check $g \uparrow a$, $g \uparrow b$, and $g = ax + by$
  - Sufficient by [MMNP11, Lemma 1]

Bipartite Graph ?

- **Input**: Undirected Graph $G=(V,E)$
- **Output**: Boolean, is the graph bipartite

- Certificate:
  - True: Partition of the vertices, $V = V_1 \cup V_2$
  - False: Odd length cycle
- Checker:
  - Verify partition or cycle

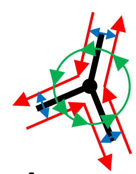
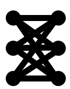
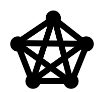
Connected Components ?

- **Input**: Undirected graph $G = (V, E)$
- **Output**: Partition of V into the c.c.
- Certificate:
  - Each vertex labeled (i, j), where i=component number, j=the nodes number in the component, such that all nodes except one in a c.c. have a neighbor with smaller j (*e.g.*, BFS numbering)
- Checker:
  - Edges connect identical i
  - Mark non-root nodes (j larger than a neighbor)
  - Check roots different labels

Shortest Path $s \rightarrow t$?

- **Input**: Directed positive weighted graph $G = (V, E)$, $s,t \in V$
- **Output**: Shortest distance $s \rightarrow t$
- Certificate:
  - Distance vector D, with distances from s to all nodes
  - Shortest path tree
- Checker:
  - Check shortest path tree implies D
  - Check that no edge can improve any distance

Planarity Graph ?

- **Input**: An undirected connected graph G
- **Output**: Boolean, is G planar
  - can *G* be drawn without edges intersecting ?
- Certificate:
  - Yes = Combinatorial Embedding  (twin edges + links + L(u))
  - No = $K_{3,3}$  or K_5  (Kuratowski subgraphs)
- Checker:
  - Yes: Check if $n+f=m+2$, n=#nodes, m=#edges, f=#boundary cycles (Euler's formula, sufficient by [MMNS11, Lemma 3])
  - No: Verify Kuratowski subgraphs

Maximum Flow ?

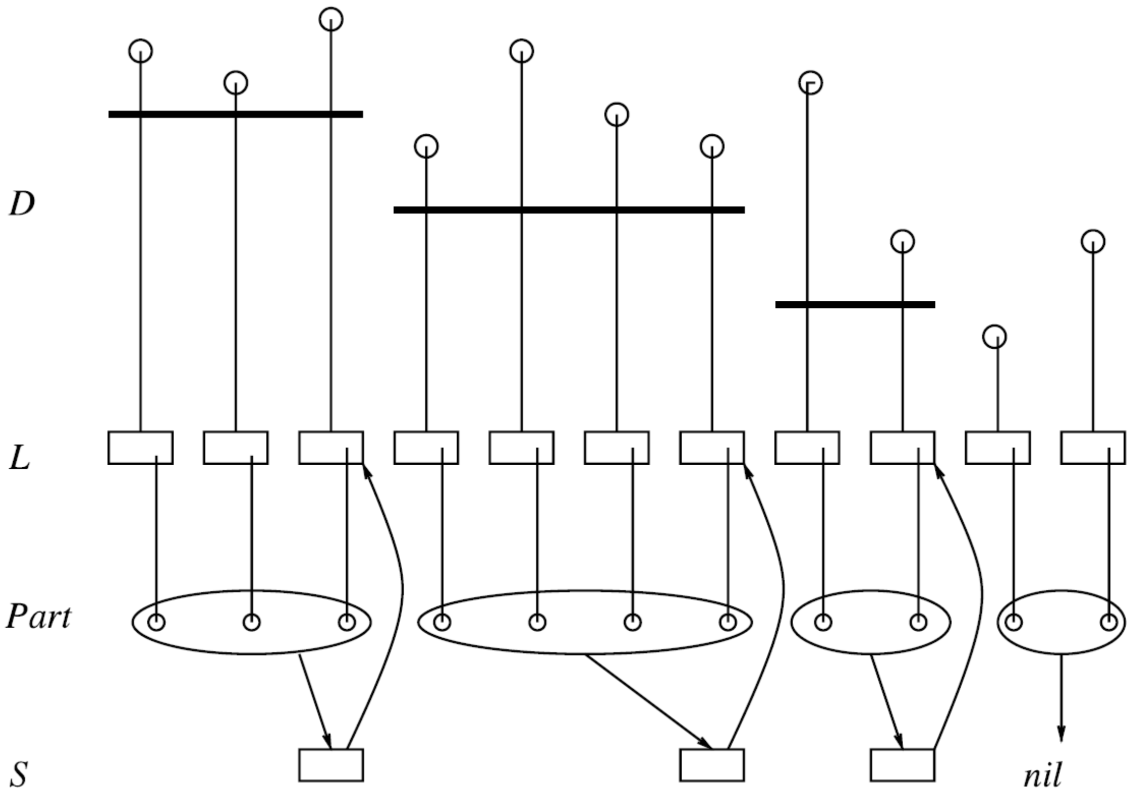
- **Input**: Flow network G, with capacity constraints c
- **Output**: Value of maximum flow

- Certificate:
  - Flow along each edge
  - Minimum cut, i.e. partition of the vertices
- Checker:
  - Check if valid flow
  - Find capacity of cut
  - Check if cut capacity is equal to value of flow

Dynamic Dictionary

- **Operations**: Insert, Delete, Search, ...
- Checker / Monitor:
  - Check maintains a doubly-linked list of *handles* into dictionary
- Checker identifies wrong queries immediately

Priority Queue



- **Operations**: Insert, DeleteMin ...
- Checker / Monitor: (see figure)
  - check element against lower bound on deletion
- Checker identifies wrong DeleteMin delayed