Constructive Logic Project **Spring 2001**	**Lars Birkedal**

Assignments

Assignments related to the course notes will be collected here throughout the course.

1. Solve the exercise mentioned on top half of page 14 of the course notes (prove that the shown rules give an alternative axiomatization of CPC).
2. Work out the detail of Example 4.2.
3. For category theorists: prove that the Kripke semantics shown on Page 19 gives the same notion of truth as that of the Kripke-Joyal semantics for the topos of presheaves on P. [Look in Mac Lane and Moedijk or in Lambek and Scott for definitions and/or if you get stuck.]
4. Do the exercise mentioned on Line 3 of Page 20.
5. Do the exercise mentioned on Lines 6–7 of Page 20.
6. Prove the remark on top of Page 21 concerning regular open sets in topological models.
7. Do the two exercises mentioned above Theorem 5.2 on Page 21.
8. Do the proof of Corollary 5.1.
9. Do the proof of Lemma 6.3.
10. Do the exercise mentioned just below the proof of Theorem 6.1.
11. Do the exercise mentioned just below the proof of Lemma 6.4.
12. Do the exercise mentioned just below the proof of Lemma 7.1.
13. Complete the proof of Lemma 7.2 (i.e., prove (AP3)).
14. Prove the statement in the footnote on Page 33.
15. Explain in words why the results on Pages 34 and 35 show that equality of real numbers is not decidable in constructive mathematics.

16. Read Troelstra and van Dalen and make sure you understand Kleene's T and U functions and his normal form theorem (Thm. 8.1 in the notes).

17. Do the exercise mentioned in the first paragraph on Page 40.

18. Explain in detail why

$$\forall n.(\exists k.T(n,n,k)) \vee (\forall k.\neg T(n,n,k))$$

is not realized. Pay particular attention to the remarks in the notes about what follows about realizability of $\forall x.P(x)$ and $\exists x.P(x)$ when $P(x)$ is primitive recursive.

19. Prove the exercise mentioned on page 40, line -15.

20. Do the remaining cases of the proof of Thm. 9.2.

21. Do the exercise concerning Church's Thesis mentioned on the bottom of Page 42.

22. Complete the proof of Lemma 9.1.

23. Prove the exercise mentioned in the proof of Corollary 9.1.

24. Prove that for A an almost negative formula, A is realized (in the non-formalized sense) iff A is true.

25. Complete the proof of Thm. 9.3.

26. Complete the proof of Thm. 9.4.

27. Do the exercise mentioned on line 3 of Page 48.

28. Do the exercise mentioned above Thm. 10.1.
