Assignment 1

Suggested Exercises from Course Notes: 1–24, 26.

Exercises to hand-in from Course Notes: 25

Extra Exercises to hand-in:

1. Show that **Rel** is a category.
   (The objects of **Rel** are sets, and an arrow $f : A \to B$ is a relation from A to B, i.e. $f \subseteq A \times B$. Composition in **Rel** is to be given by:

   $$g \circ f = \{\langle a, c\rangle \in A \times C \mid \exists b\,(\langle a, b\rangle \in f \;\&\; \langle b, c\rangle \in g)\}$$

   for $f \subseteq A \times B$ and $g \subseteq B \times C$).

2. Consider the following isomorphisms of categories and determine which hold.

   (a) $\mathbf{Rel} \cong \mathbf{Rel}^{\mathrm{op}}$

   (b) $\mathbf{Sets} \cong \mathbf{Sets}^{\mathrm{op}}$

   (c) For a set X, $P(X) \cong P(X)^{\mathrm{op}}$ (as poset categories)

3. (a) Show that in **Sets**, the isomorphisms are exactly the bijections.

   (b) Show that in **Monoids**, the isomorphisms are exactly the bijective homomorphisms.

   (c) Show that in **Posets**, the isomorphisms are *not* just the bijective homomorphisms.