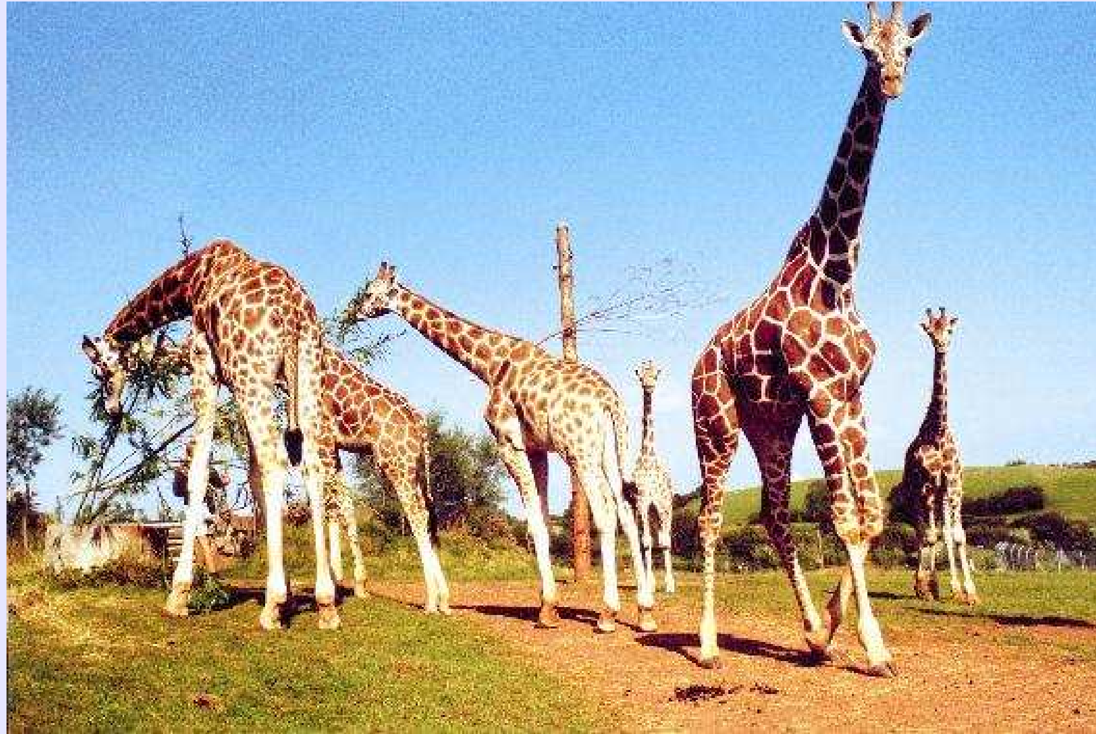
Cache-Oblivious String Dictionaries



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Joint work with Rolf Fagerberg

ADS 2005 – 2nd Bertinoro Workshop on Algorithms and Data Structures
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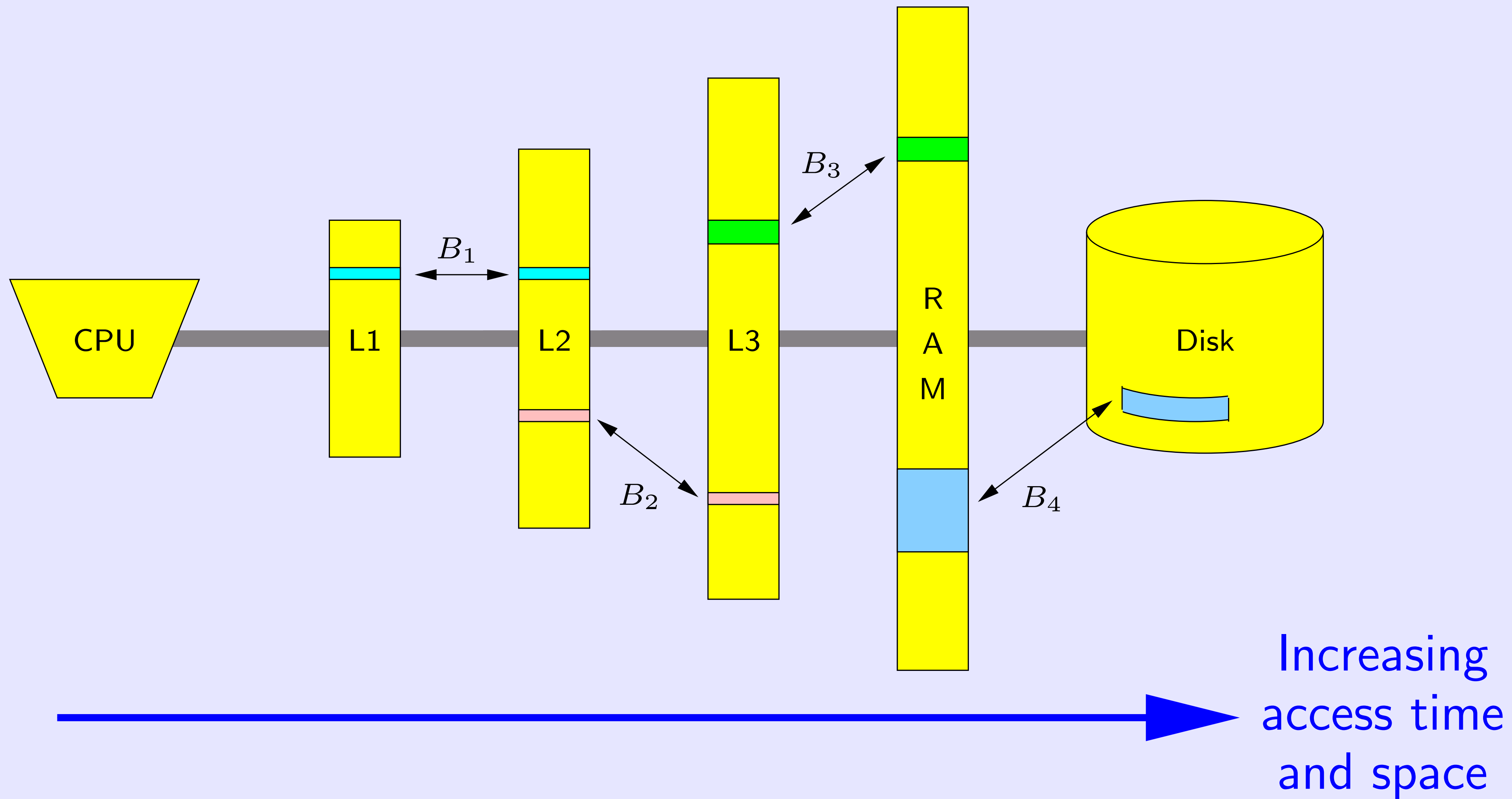
Outline of Talk

- Cache-oblivious model
- Basic cache-oblivious techniques
- Cache-oblivious string algorithms
- Cache-oblivious string dictionaries
  - Cache-oblivious tries and blind tries

Hierarchical Memory Models

Hierarchical Memory



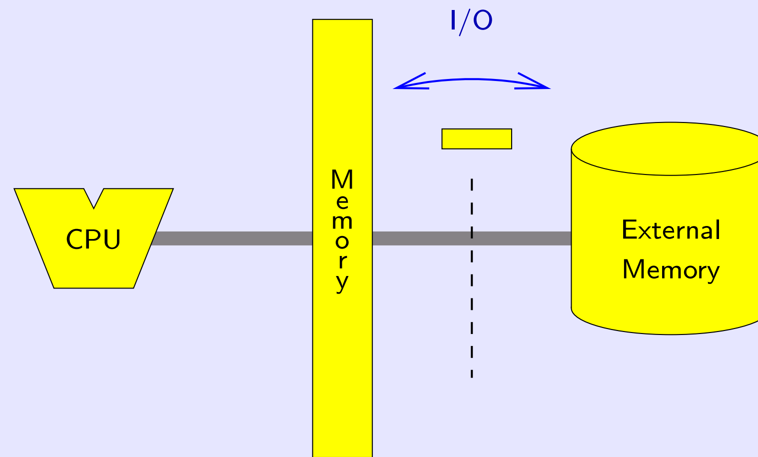
Hierarchical Memory

	Latency	Relative to CPU
Register	0.5 ns	1
L1 cache	0.5 ns	1-2
L2 cache	3 ns	2-7
DRAM	150 ns	80-200
TLB	500+ ns	200-2000
Disk	10 ms	10^7

- Accessing non-local storage may take a very long time
- Good locality is important for achieving high performance

I/O Model

Aggarwal and Vitter 1988



N = problem size

M = memory size

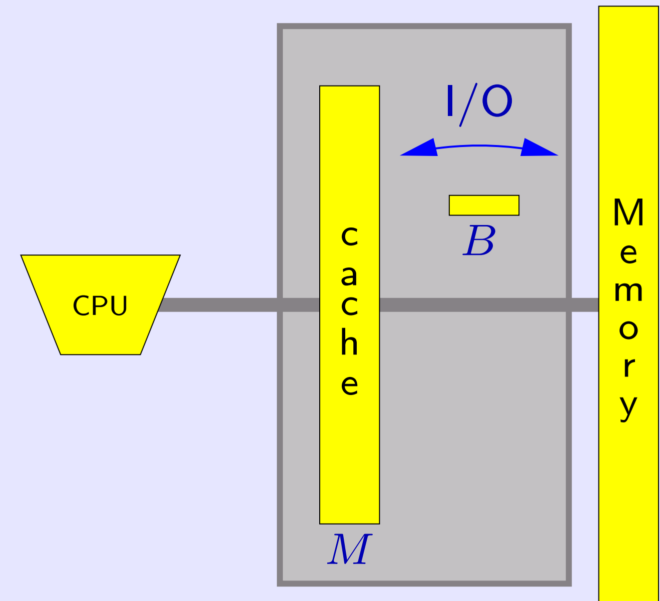
B = I/O block size

- One I/O moves B consecutive records from/to disk
- Complexity measure = number of I/Os

Ideal Cache Model — no parameters!?

Frigo, Leiserson, Prokop, Ramachandran 1999

- Program with only one memory
- Analyze in the I/O model for
- Optimal off-line cache replacement strategy arbitrary B and M



Ideal Cache Model — no parameters!?

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- Program with only one memory
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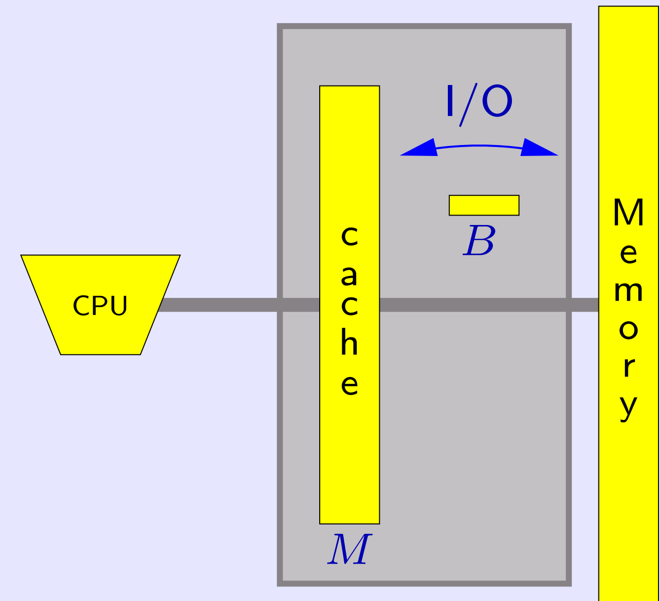
- Optimal off-line cache replacement strategy arbitrary B and M

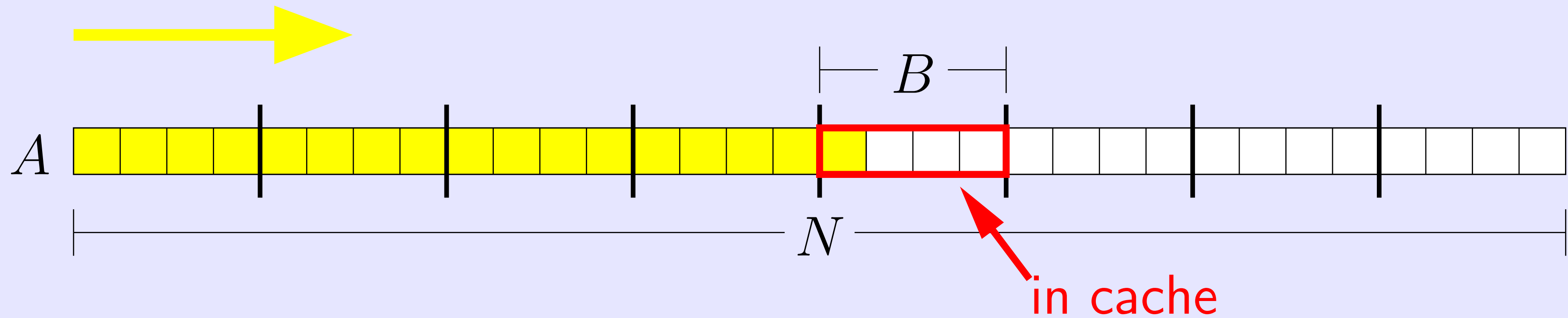


Advantages

- Optimal on arbitrary level $\Rightarrow$ optimal on all levels
- Portability, B and M not hard-wired into algorithm
- Dynamic changing M (and B)

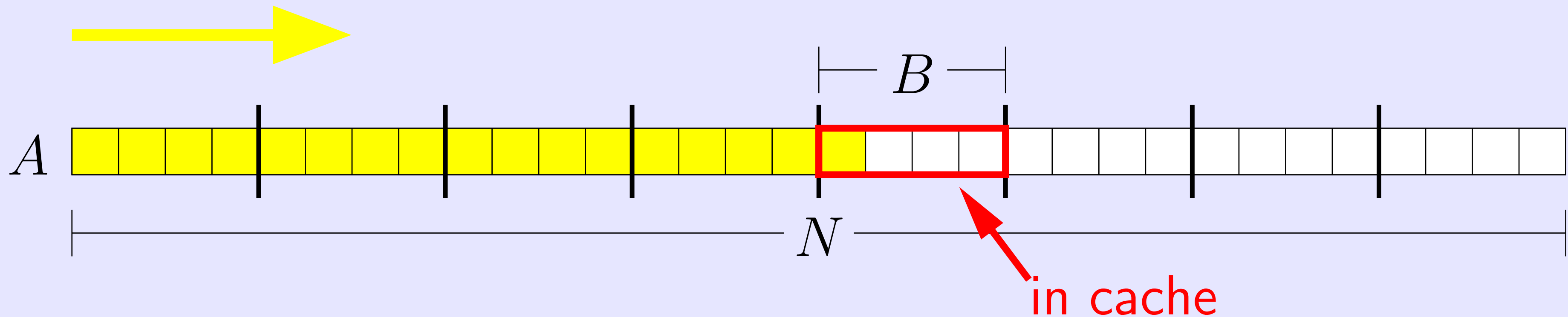
Cache-Oblivious Preliminaries

Cache-Oblivious Scanning



$$O\left(\frac{N}{B}\right) \text{ I/Os}$$

Cache-Oblivious Scanning

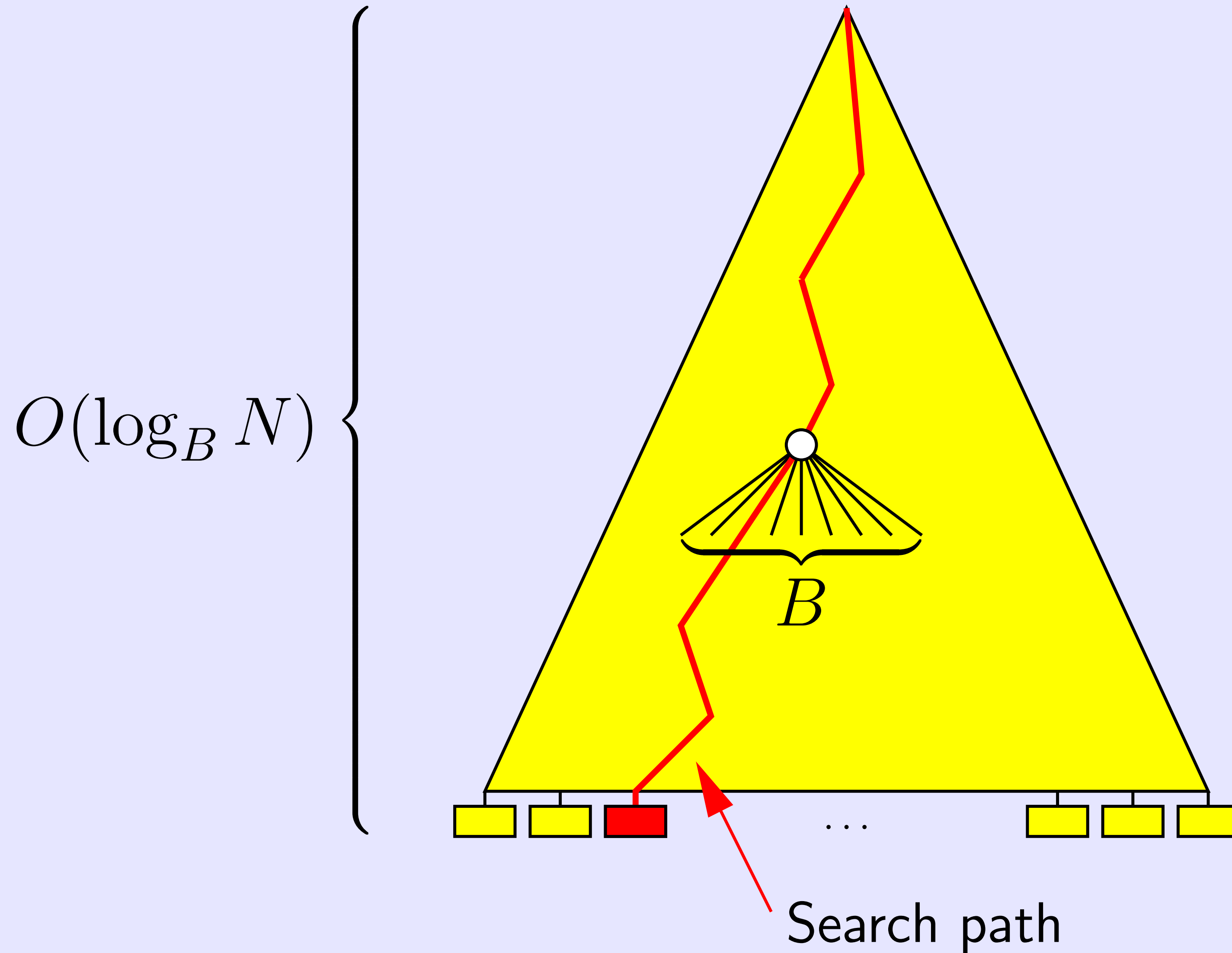


$$O\left(\frac{N}{B}\right) \text{ I/Os}$$

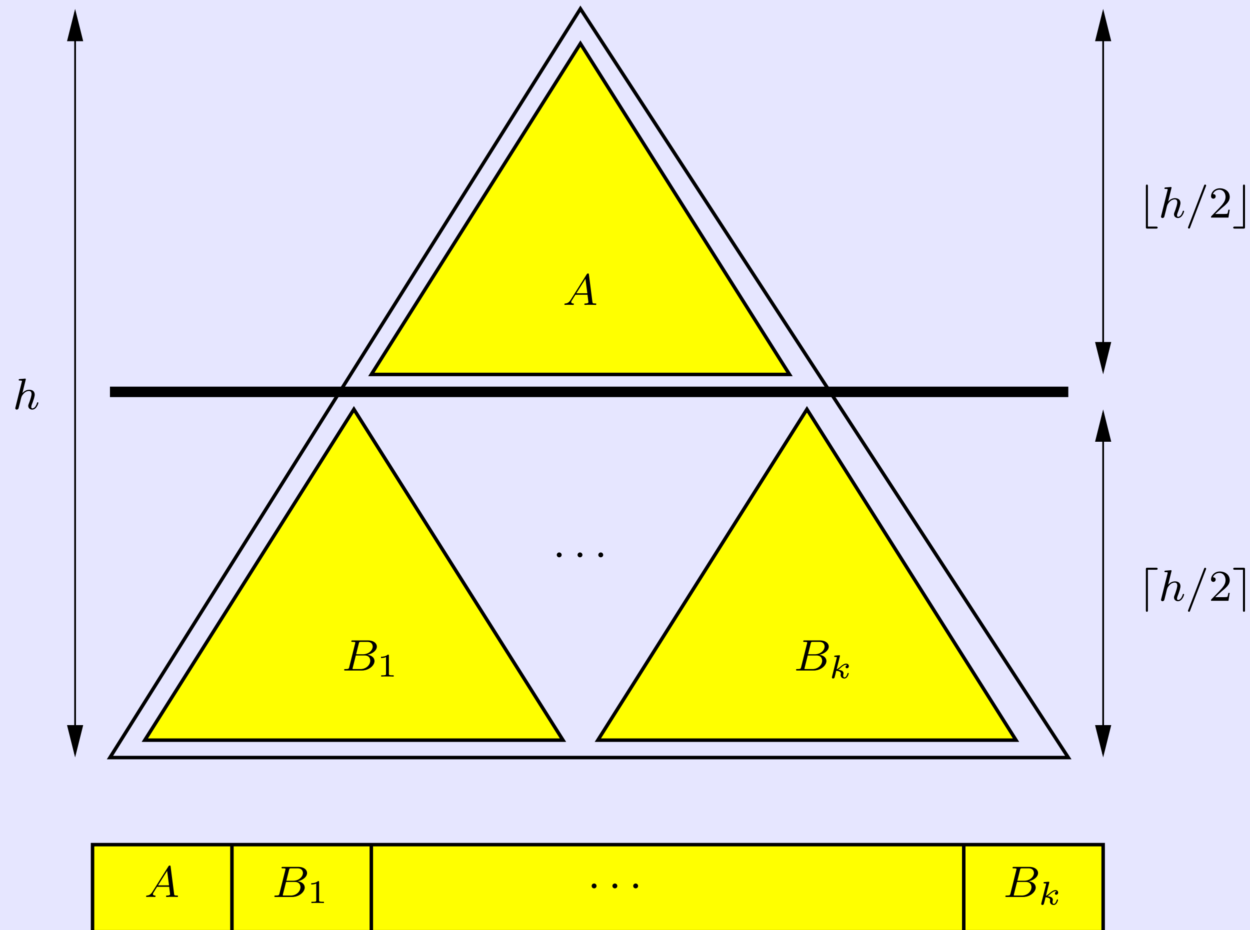
Corollary Cache-oblivious selection requires $O(N/B)$ I/Os

Hoare 1961 / Blum et al. 1973

Cache-Aware B-trees

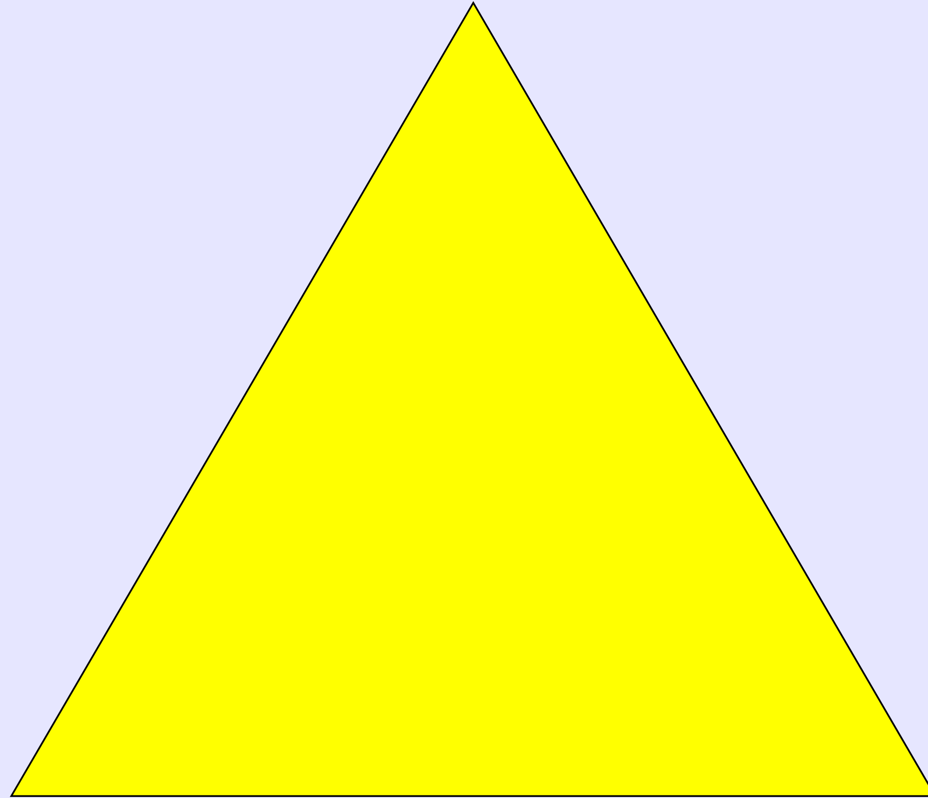


Static Cache-Oblivious B-Tree

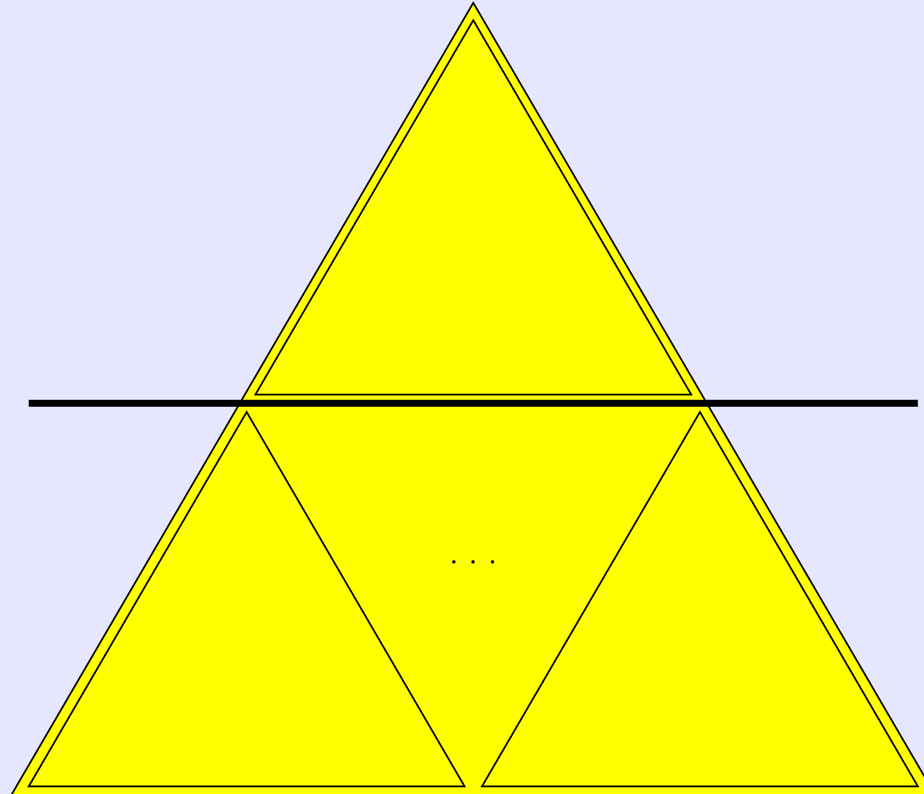


Recursive layout of binary tree $\equiv$ van Emde Boas layout

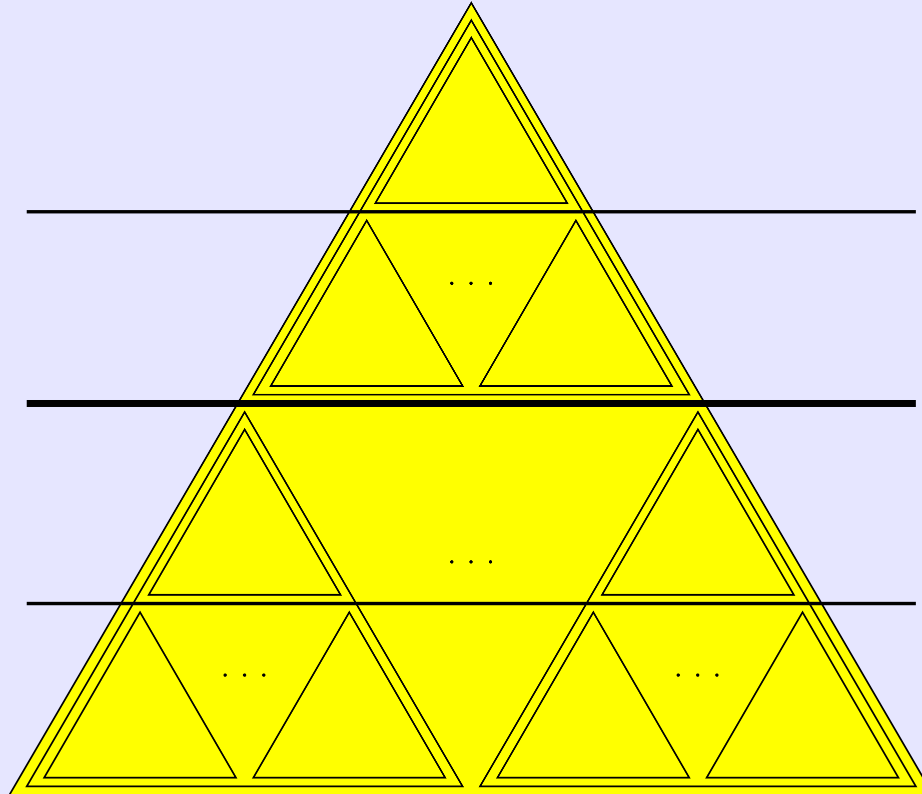
Static Cache-Oblivious B-Tree



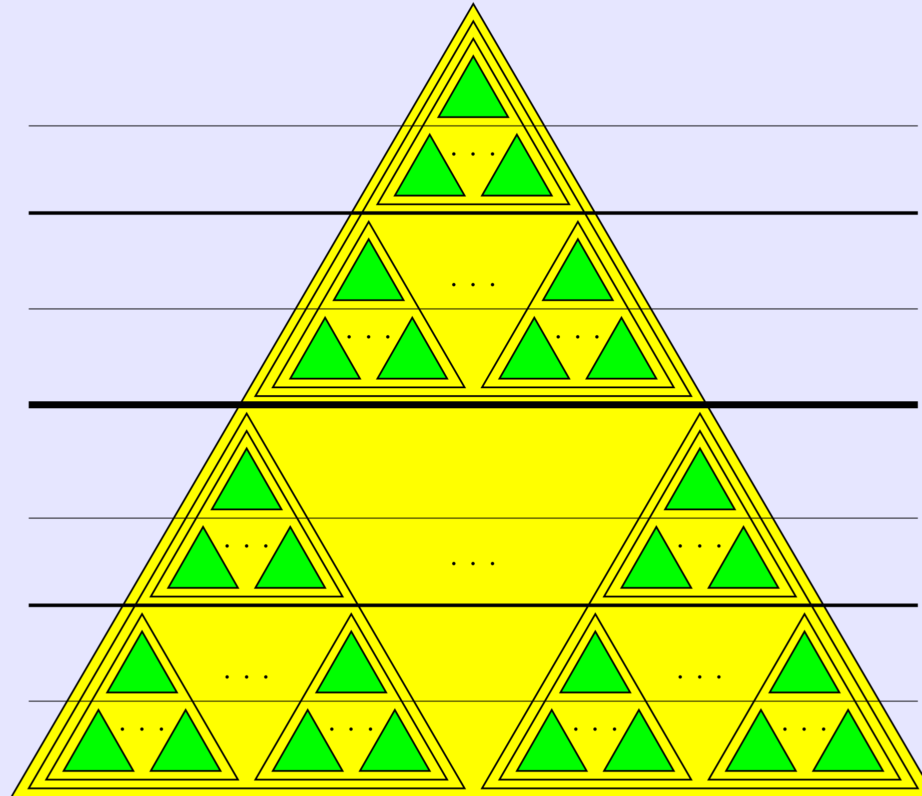
Static Cache-Oblivious B-Tree



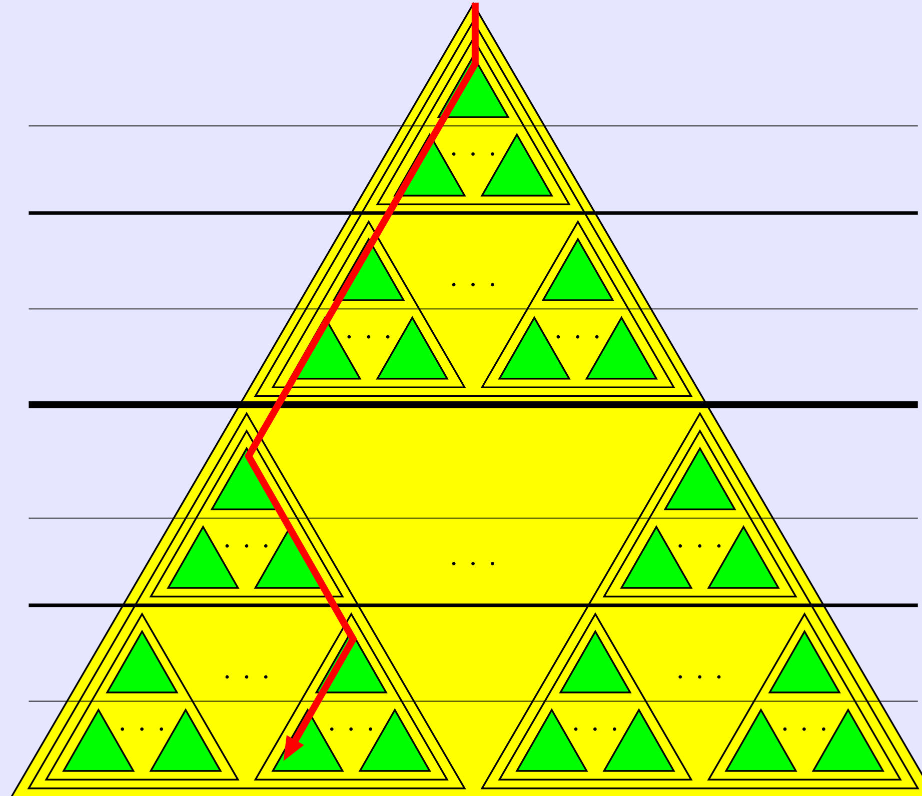
Static Cache-Oblivious B-Tree



Static Cache-Oblivious B-Tree

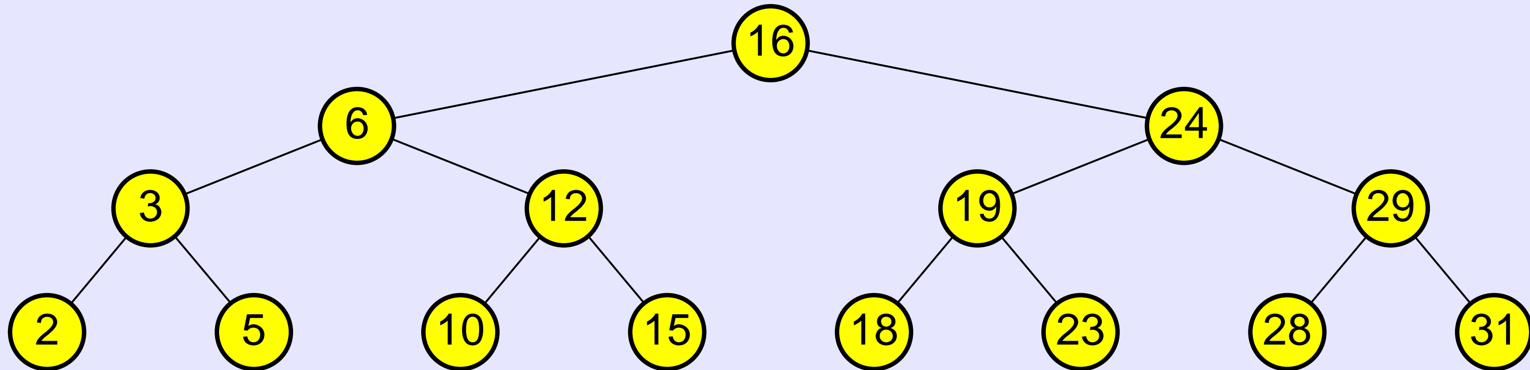


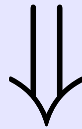
Static Cache-Oblivious B-Tree



- Each green tree has height between $(\log_2 B)/2$ and $\log_2 B$
- Searches visit between $\log_B N$ and $2 \log_B N$ green trees, i.e. perform at most $4 \log_B N$ I/Os (misalignment)

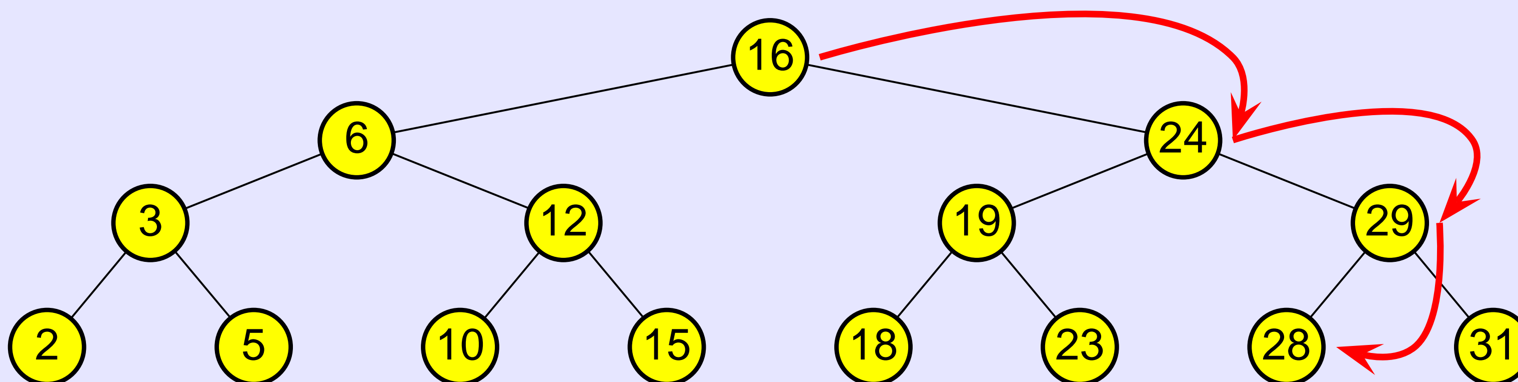
Example : Recursive Layout

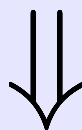


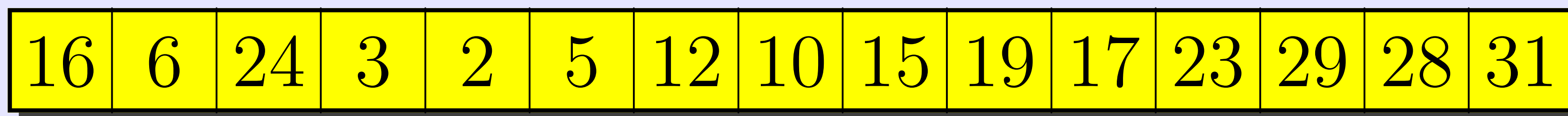


16	6	24	3	2	5	12	10	15	19	17	23	29	28	31

Example : Recursive Layout







Summary Cache-Oblivious Tools

Scanning :	$O(N/B)$
B-tree searching :	$O(\log_B N)$
Sorting* :	$O\left(\frac{N}{B}\log_{M/B}\frac{N}{B}\right)$

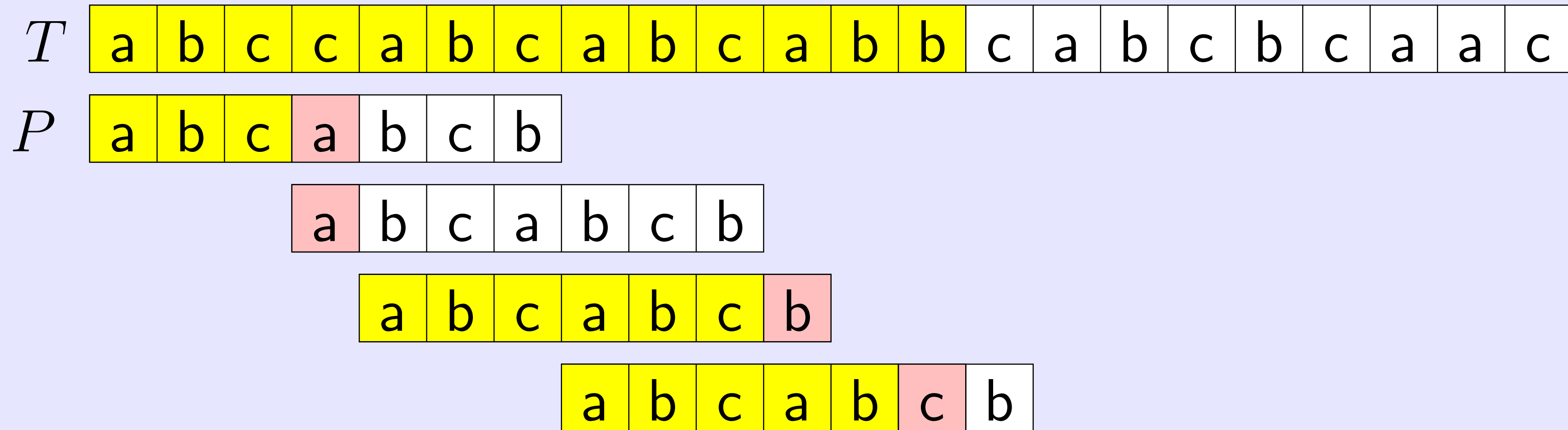
* requires a tall cache assumption $M \geq B^{1+\varepsilon}$

Frigo, Leiserson, Prokop, Ramachandran 1999
Brodal and Fagerberg 2002, 2003

Cache-Oblivious String Algorithms

Knuth-Morris-Pratt String Matching

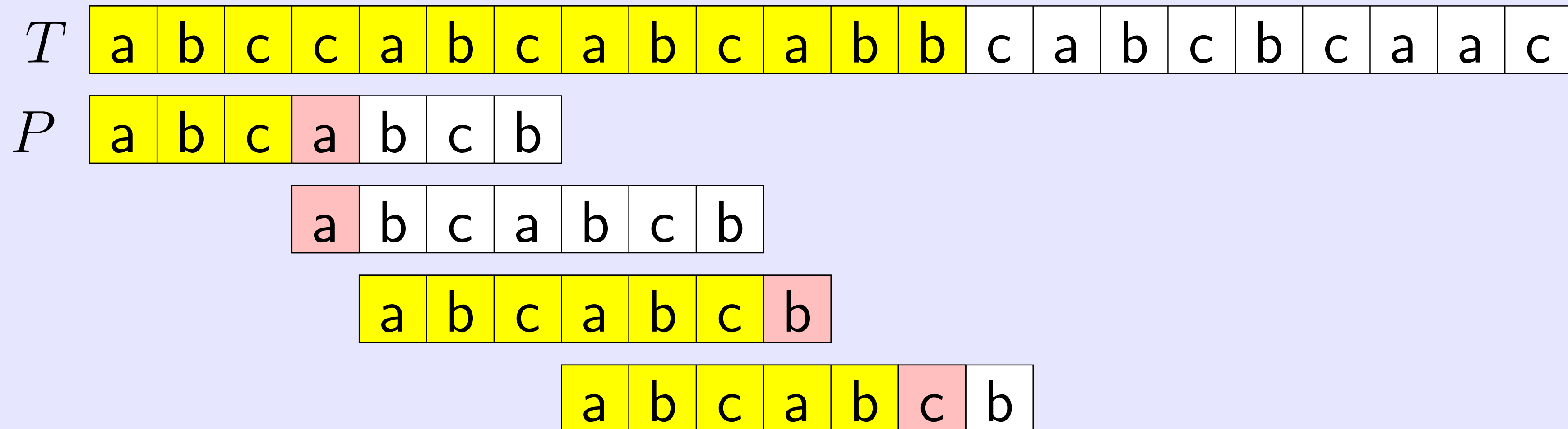
Knuth, Morris, Pratt 1977



- Time $O(|T|)$
- Scans text left-to-right
- Accesses the pattern (and failure function) like a stack

Knuth-Morris-Pratt String Matching

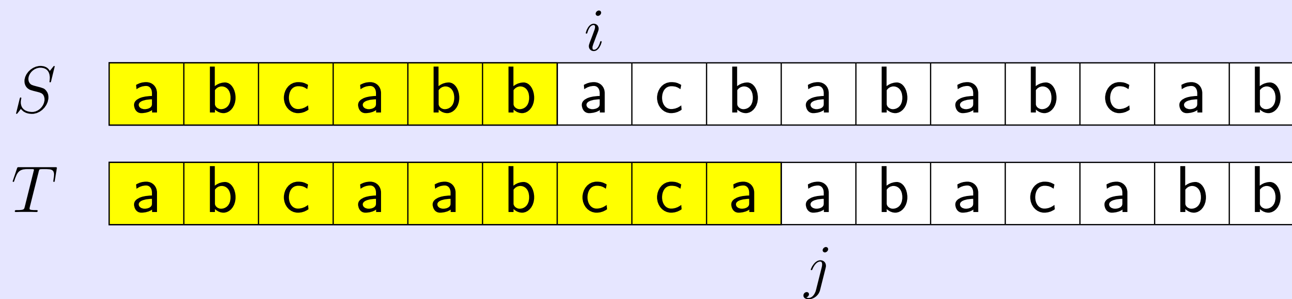
Knuth, Morris, Pratt 1977



- Time $O(|T|)$
- Scans text left-to-right
- Accesses the pattern (and failure function) like a stack
- KMP is cache-oblivious and uses $O(|T|/B)$ I/Os

Edit Distance

$$E(i,j) = \begin{cases} i & \text{if} \quad j = 0 \\ j & \text{if} \quad i = 0 \\ E(i-1, j-1) & \text{if} \quad S[i] = T[j] \\ 1 + \min\{E(i-1,j), E(i,j-1)\} & \text{if} \quad S[i] \neq T[j] \end{cases}$$



- Dynamic programming
- Time $O(|S| \cdot |T|)$
- Space $O(\min\{|S|, |T|\})$

Edit Distance

$E(i,j)$	0	1	2	3			$\cdots$			j				$\cdots$			$\lvert T\rvert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
$\vdots$	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
$\vdots$	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\lvert S\rvert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

Result

Edit Distance

$E(i,j)$	0	1	2	3			$\cdots$			j			$\cdots$				$\lvert T\rvert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
$\vdots$	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
$\vdots$	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\lvert S\rvert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

One array of size $|T|+2$

Result

Row-by-row computation requires $O\left(\frac{|S|\cdot|T|}{B}\right)$ I/Os

Recursive Edit Distance

$E(i,j)$	0	1	2	3			$\cdots$			j			$\cdots$				$\lvert T\rvert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
$\vdots$	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
$\vdots$	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\lvert S\rvert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

array of size $|S|+|T|+1$

Result

Recursive Edit Distance

$E(i,j)$	0	1	2	3			$\cdots$			j			$\cdots$				$\lvert T\rvert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
$\vdots$	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
$\vdots$	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\lvert S\rvert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

array of size $|S| + |T| + 1$

Result

Recursive Edit Distance

$E(i,j)$	0	1	2	3			$\cdots$			j				$\cdots$			$\lvert T\rvert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
$\vdots$	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
$\vdots$	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\lvert S\rvert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

array of size $|S| + |T| + 1$

Result

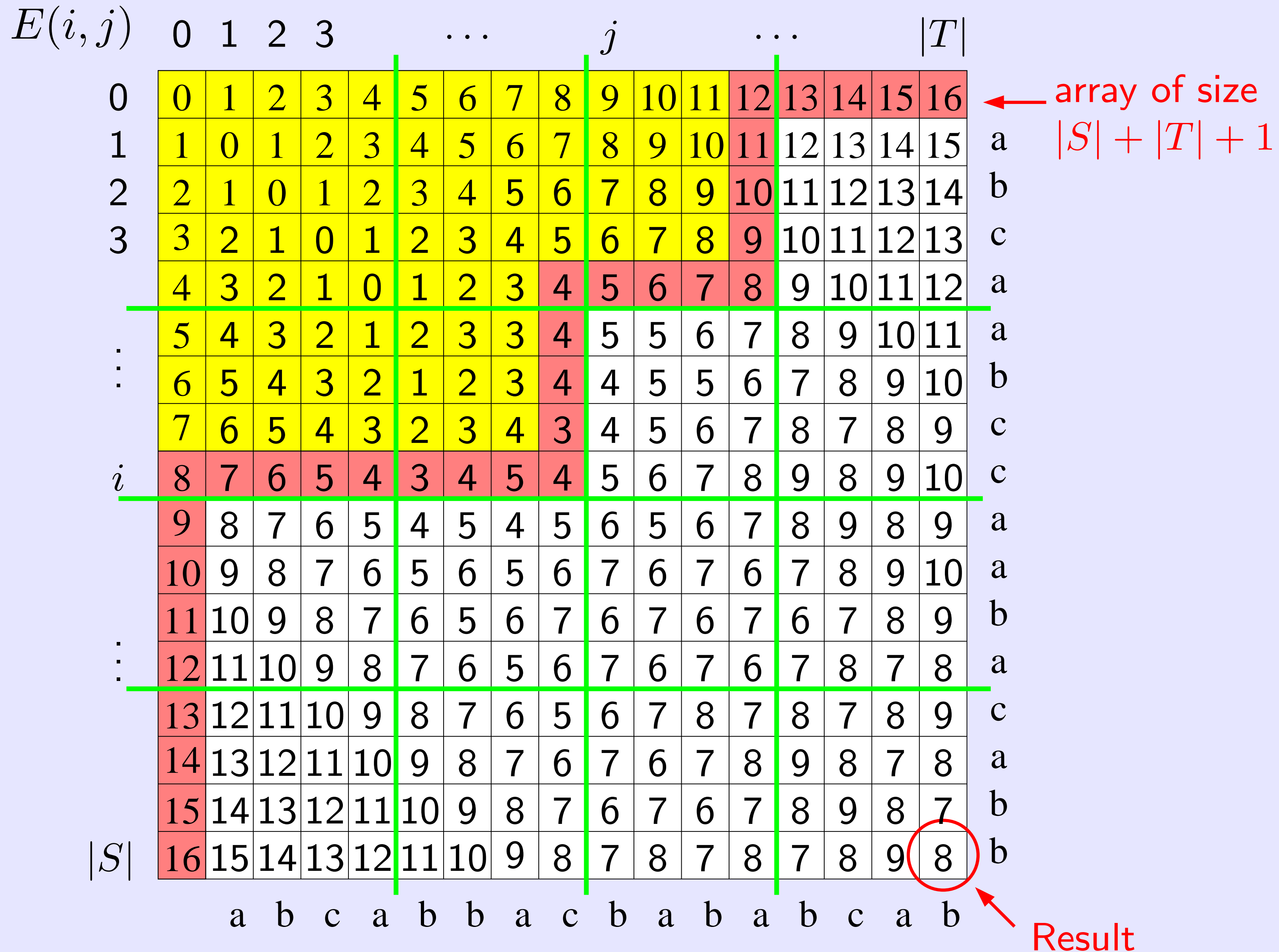
Recursive Edit Distance

$E(i,j)$	0	1	2	3			$\cdots$			j				$\cdots$			$\lvert T\rvert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
$\vdots$	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
$\vdots$	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\lvert S\rvert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

array of size $|S| + |T| + 1$

Result

Recursive Edit Distance



$E(i,j)$	0	1	2	3			$\cdots$			j				$\cdots$			$\vert T\vert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
$\vdots$	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
$\vdots$	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\vert S\vert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

Recursive Edit Distance

$E(i,j)$	0	1	2	3			$\cdots$			j			$\cdots$				$\lvert T\rvert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
$\vdots$	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
$\vdots$	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\lvert S\rvert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

array of size $|S| + |T| + 1$

Result

Recursive Edit Distance

$E(i,j)$	0	1	2	3			$\cdots$			j				$\cdots$			$\lvert T\rvert$	
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	a
2	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b
3	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	c
	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	a
$\vdots$	5	4	3	2	1	2	3	3	4	5	5	6	7	8	9	10	11	a
	6	5	4	3	2	1	2	3	4	4	5	5	6	7	8	9	10	b
	7	6	5	4	3	2	3	4	3	4	5	6	7	8	7	8	9	c
i	8	7	6	5	4	3	4	5	4	5	6	7	8	9	8	9	10	c
	9	8	7	6	5	4	5	4	5	6	5	6	7	8	9	8	9	a
	10	9	8	7	6	5	6	5	6	7	6	7	6	7	8	9	10	a
	11	10	9	8	7	6	5	6	7	6	7	6	7	6	7	8	9	b
$\vdots$	12	11	10	9	8	7	6	5	6	7	6	7	6	7	8	7	8	a
	13	12	11	10	9	8	7	6	5	6	7	8	7	8	7	8	9	c
	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	8	a
	15	14	13	12	11	10	9	8	7	6	7	6	7	8	9	8	7	b
$\lvert S\rvert$	16	15	14	13	12	11	10	9	8	7	8	7	8	7	8	9	8	b
		a	b	c	a	b	b	a	c	b	a	b	a	b	c	a	b	

array of size $|S| + |T| + 1$

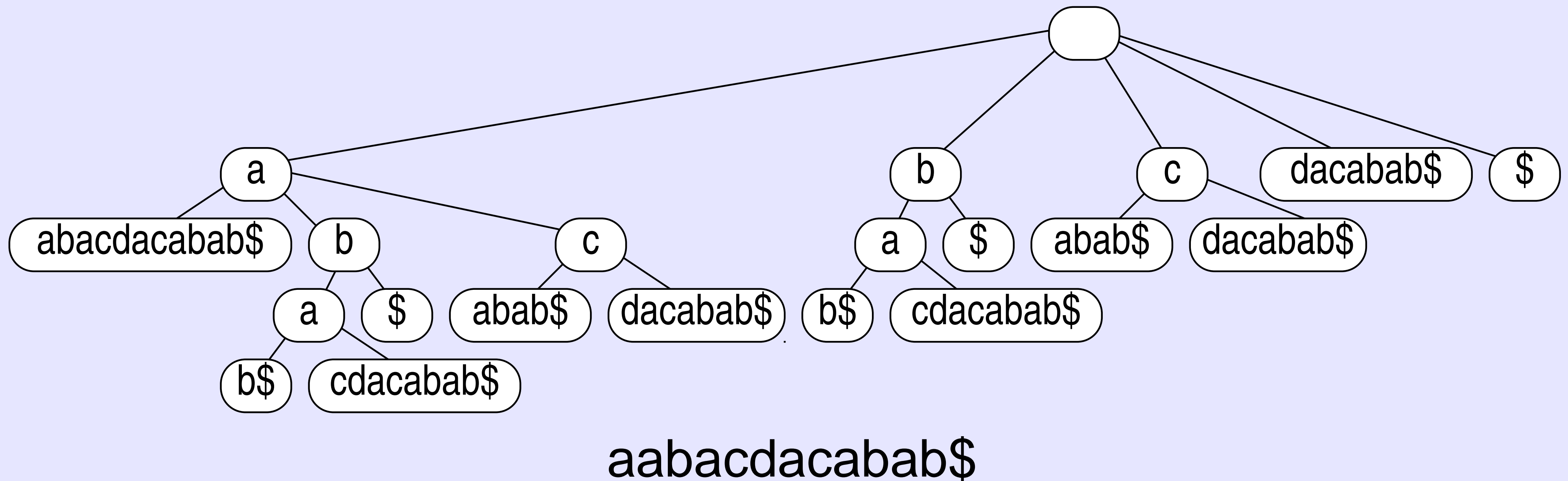
$M \times M$

Result

Recursive computation requires $O\left(\frac{|S|\cdot|T|}{M^2} \cdot \frac{M}{B}\right) = O\left(\frac{|S|\cdot|T|}{M\cdot B}\right)$ I/Os

Suffix Tree/Suffix Array Construction

Farach et al. 2000



aabacdacabab$

- Reduces to sorting, i.e. $\mathrm{Sort}(N)$ I/Os

Sorting n Strings of Total Length N

- Internal memory $O(n \log n + N)$ time
- The strings can be sorted using suffix tree construction,

  $\{$ acabab , aabac, bac $\}$ $\Rightarrow$ aabac#acabab#bac$

  i.e. cache-oblivious and $\mathrm{Sort}(N)$ I/Os
- Cache-aware Arge et al. 1997

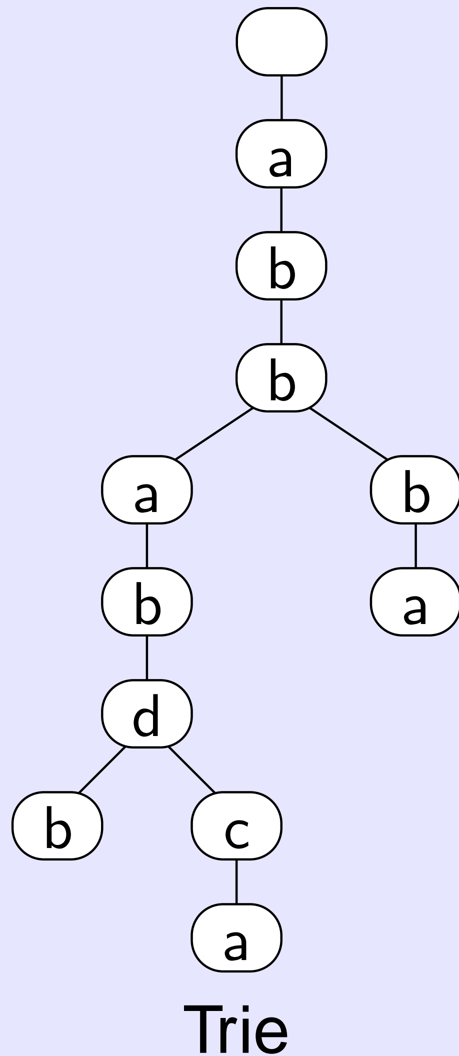
  $O\left(\min\left\{\frac{N_1}{B}\log_{M/B}\frac{N_1}{B}, K_1\log_M K_1\right\} + K_2\log_M K_2 + \frac{N}{B}\right)$ I/Os

  K_1 short strings (length $\leq B$) with total length N_1

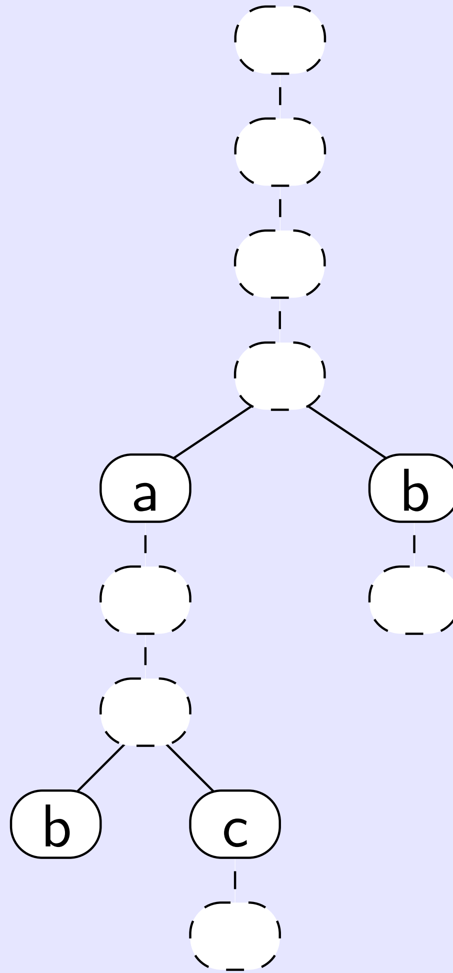
  K_2 long strings with total length N_2

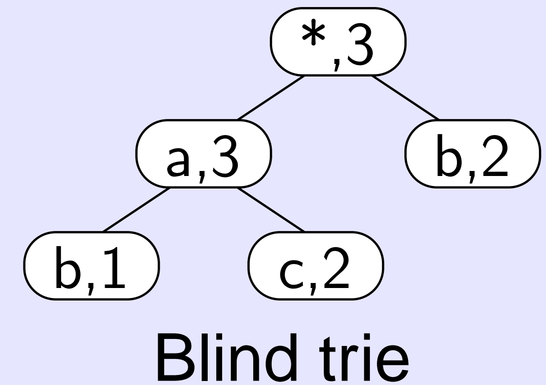
String Dictionaries

Tries vs Blind Tries



Trie





Blind trie

Searches take $O(|P|)$ time in internal memory for constant sized alphabets and $O(\log n + |P|)$ time for comparison based alphabets

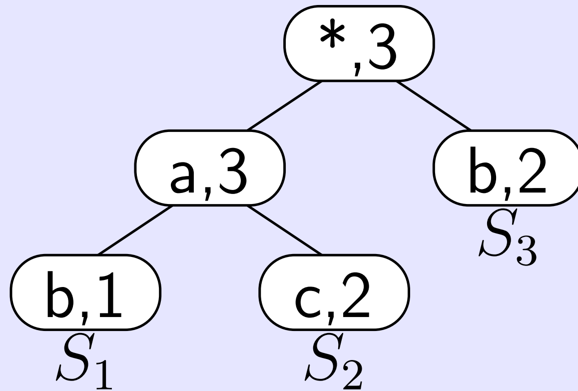
The Trouble Starts...

– Tries cannot be stored cache-aware to support top-down searches in $O(\log_B N + |P|/B)$ I/Os — Demaine et al 2004

– Can construct suffix trees cache-obliviously using $O(\mathrm{Sort}(N))$ I/Os, but cannot search in it efficiently...

+ Cache-aware string B trees support searches in a set of strings in $O(\log_B n + |P|/B)$ I/Os — Ferragina and Grossi 1999

String Dictionary



S_1 | a | b | b | a | b | d | b |

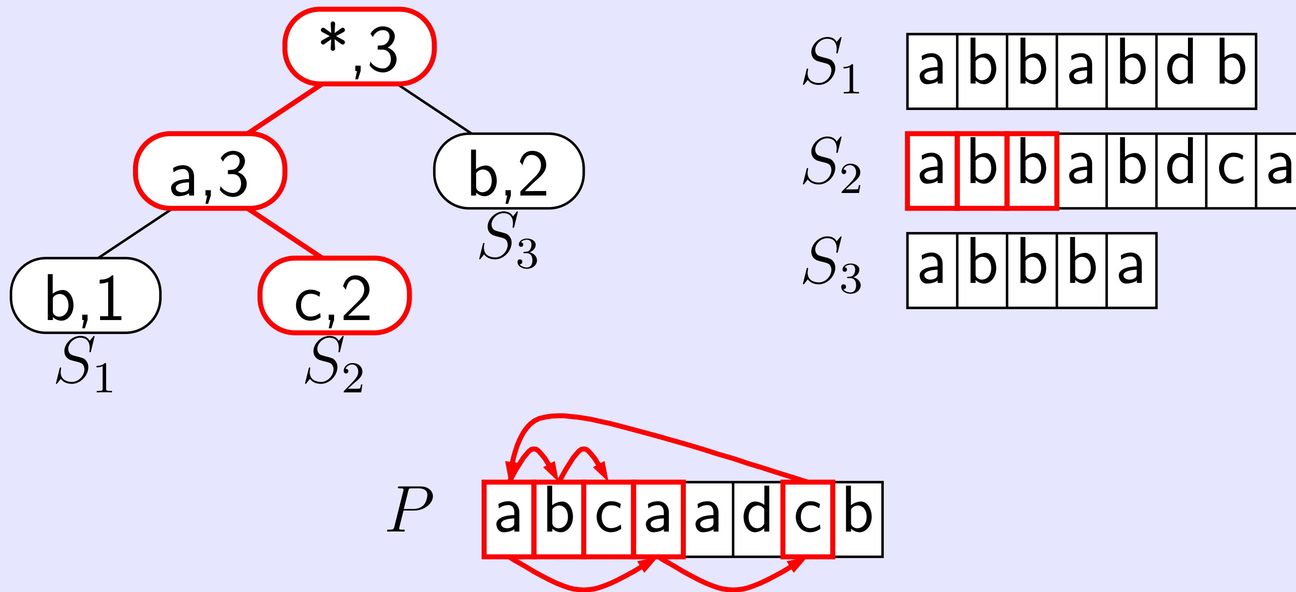
S_2 | a | b | b | a | b | d | c | a |

S_3 | a | b | b | b | a |

P | a | b | c | a | a | d | c | b |

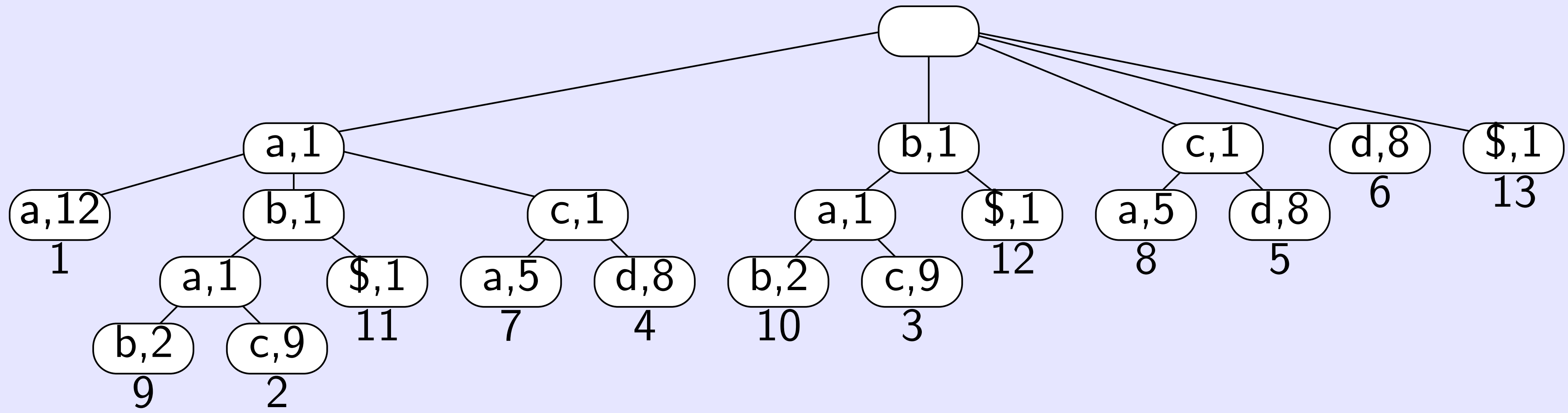
Queries: Search blind trie + Verify one string

String Dictionary



Queries: Search blind trie + Verify one string

Suffix Tree



T

a	a	b	a	c	d	a	c	a	b	a	b	$

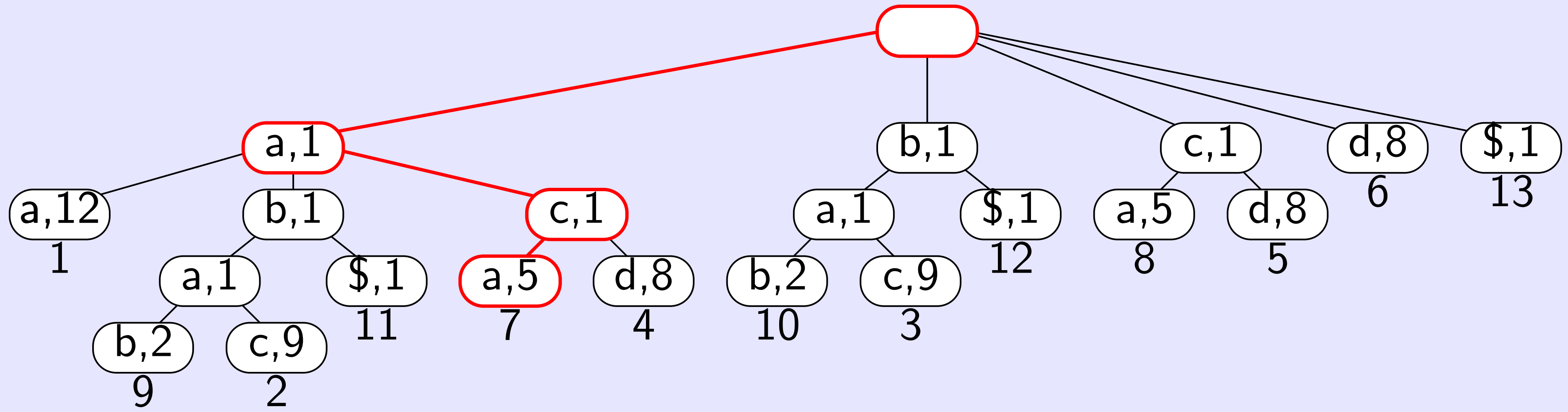
1 2 $\cdots$ 7 $\cdots$ 13

P

a	c	a	d	a

Queries: Search blind trie + Verify one suffix

Suffix Tree

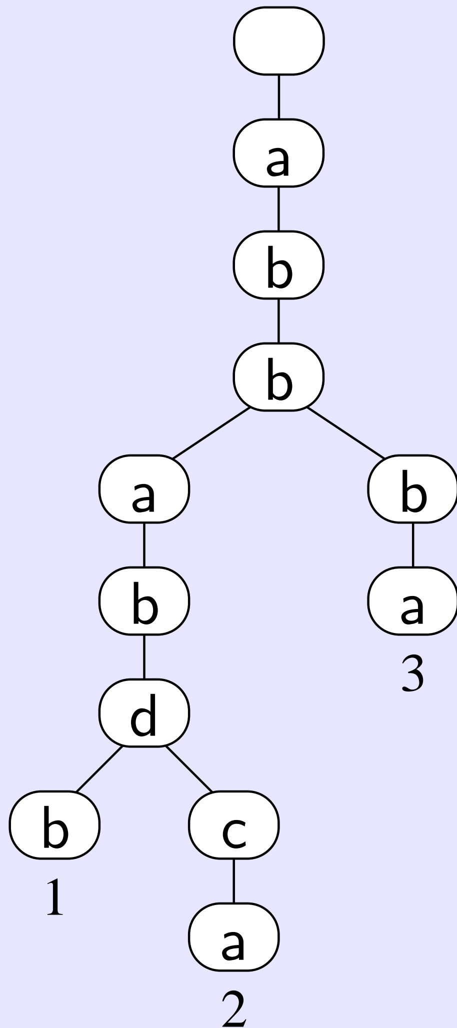


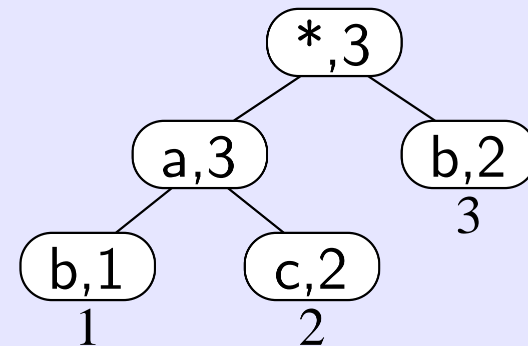
T	a	a	b	a	c	d	a	c	a	b	a	b	$
	1	2	$\cdots$				7		$\cdots$				13

P	a	c	a	d	a

Queries: Search blind trie + Verify one suffix

Tries

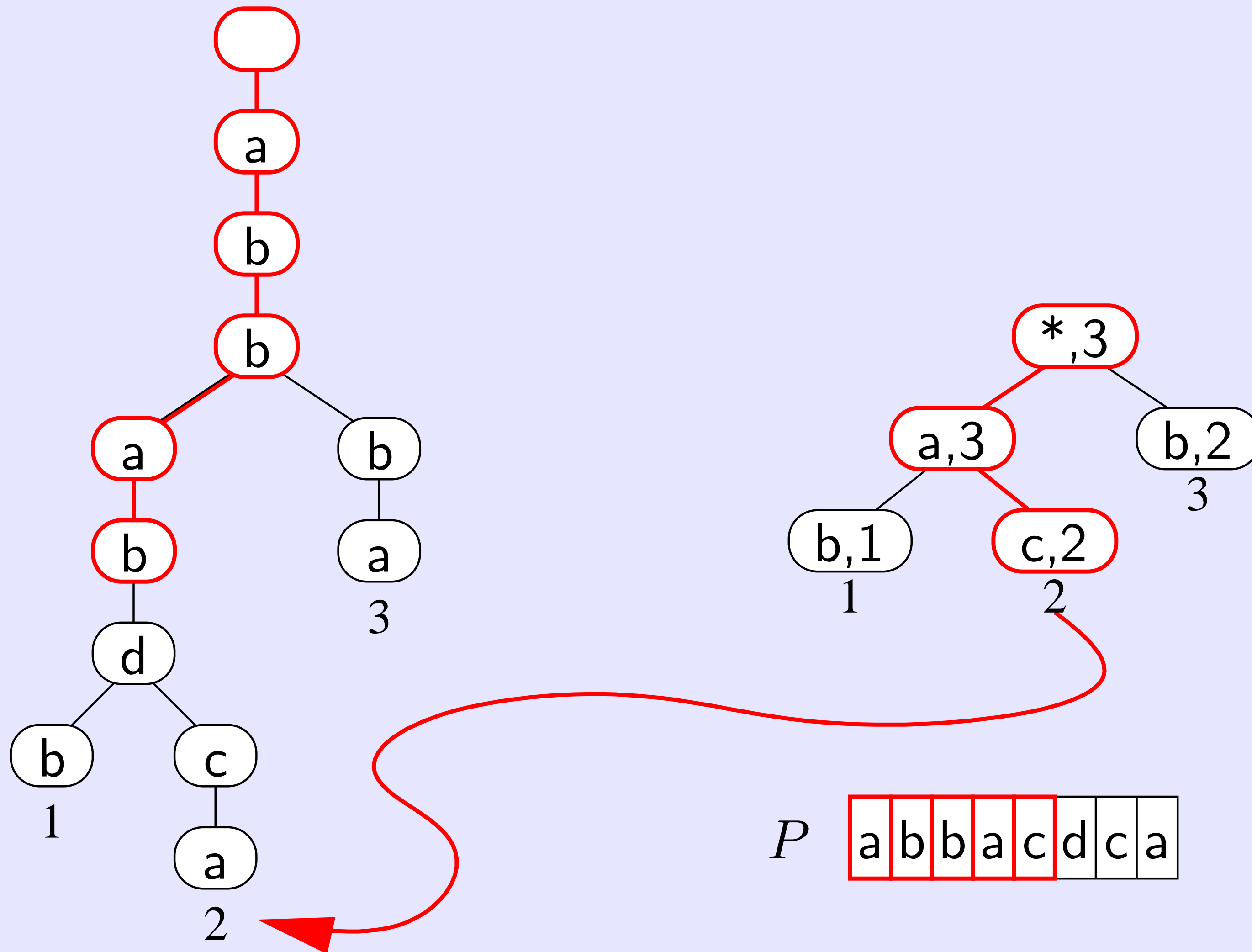




P | a | b | b | a | c | d | c | a |

Queries: Search blind trie + Verify prefix of one path

Tries



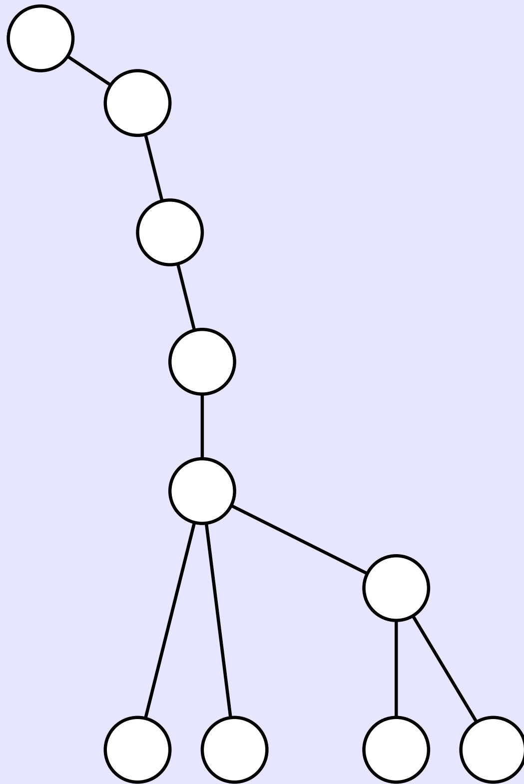
Queries: Search blind trie + Verify prefix of one path

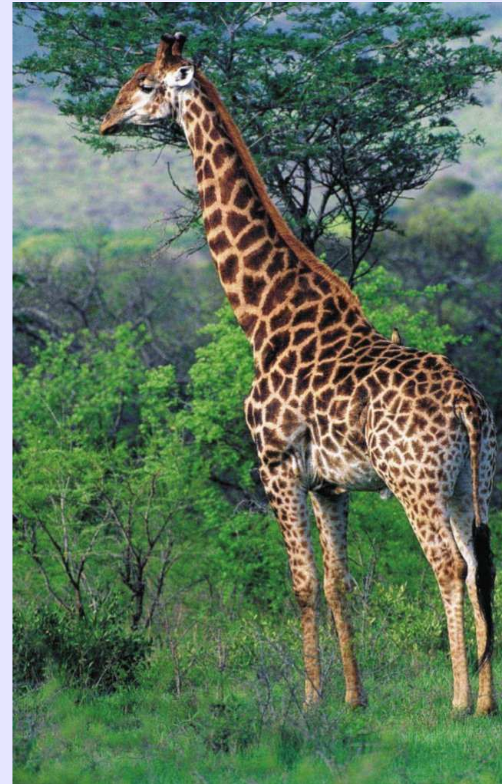
Verifying a Prefix of a Path in a Tree

Verifying Paths in Giraffe Trees is Easy

Definition

A tree is a giraffe tree if all root-to-leaf paths share at least half of the nodes of the tree (long neck)





Verifying Paths in Giraffe Trees is Easy

Definition

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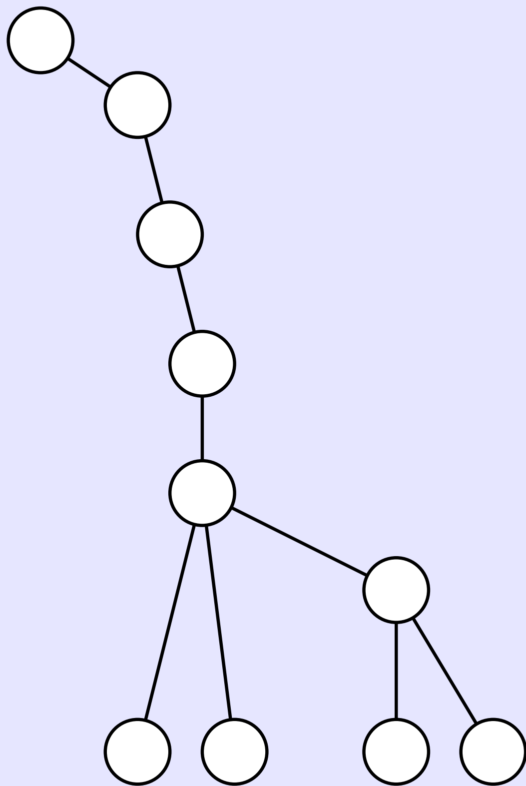
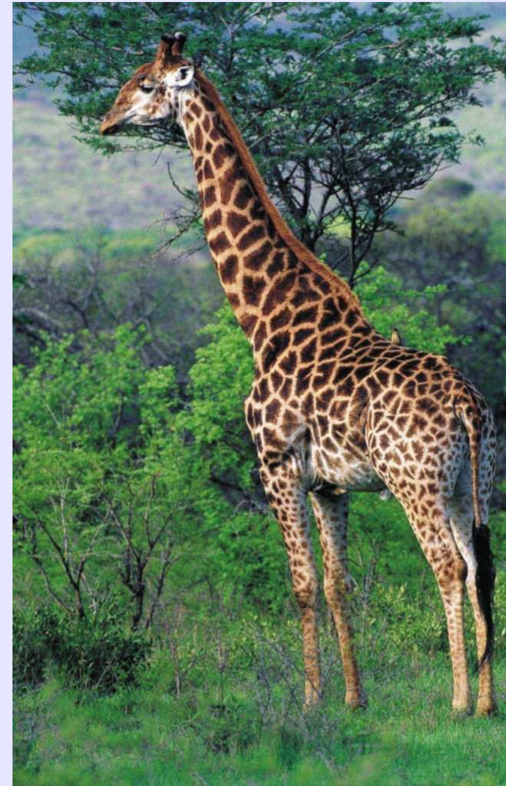
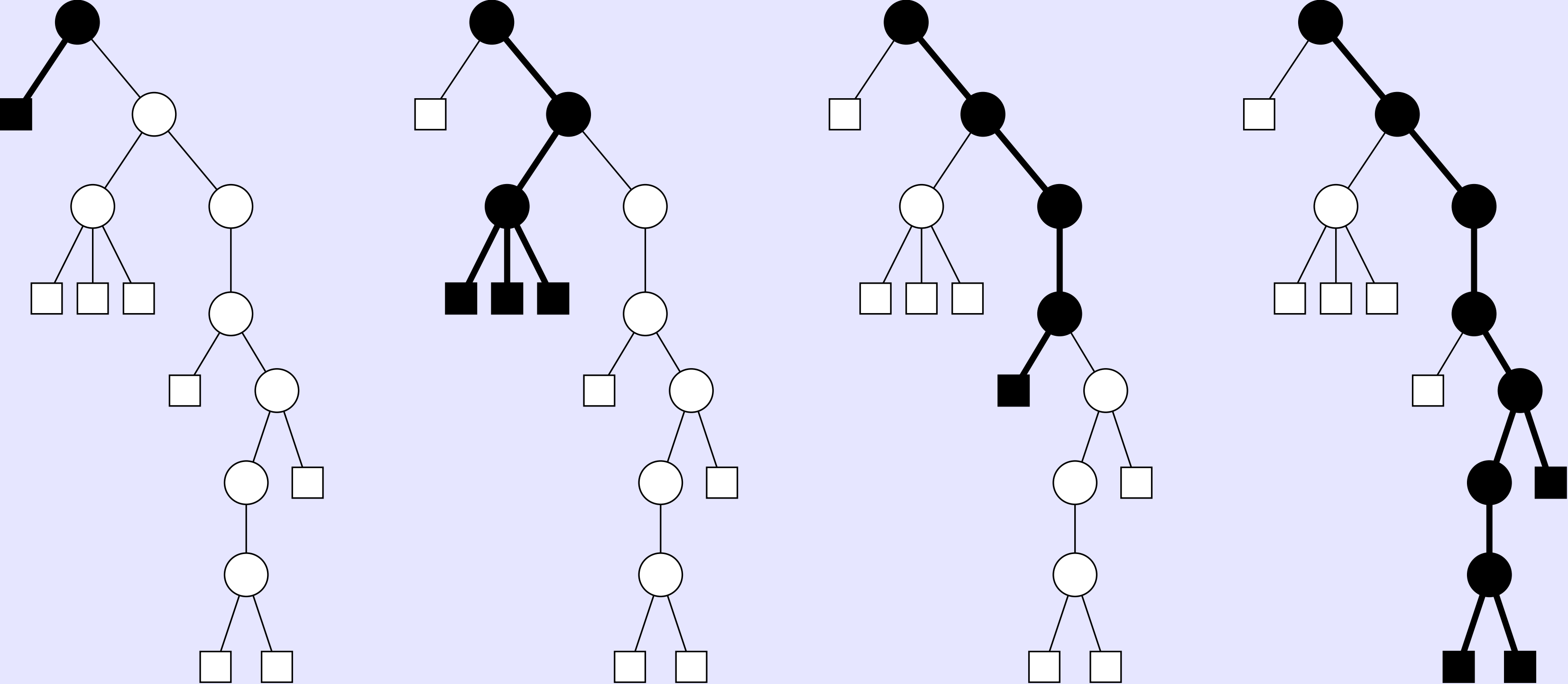


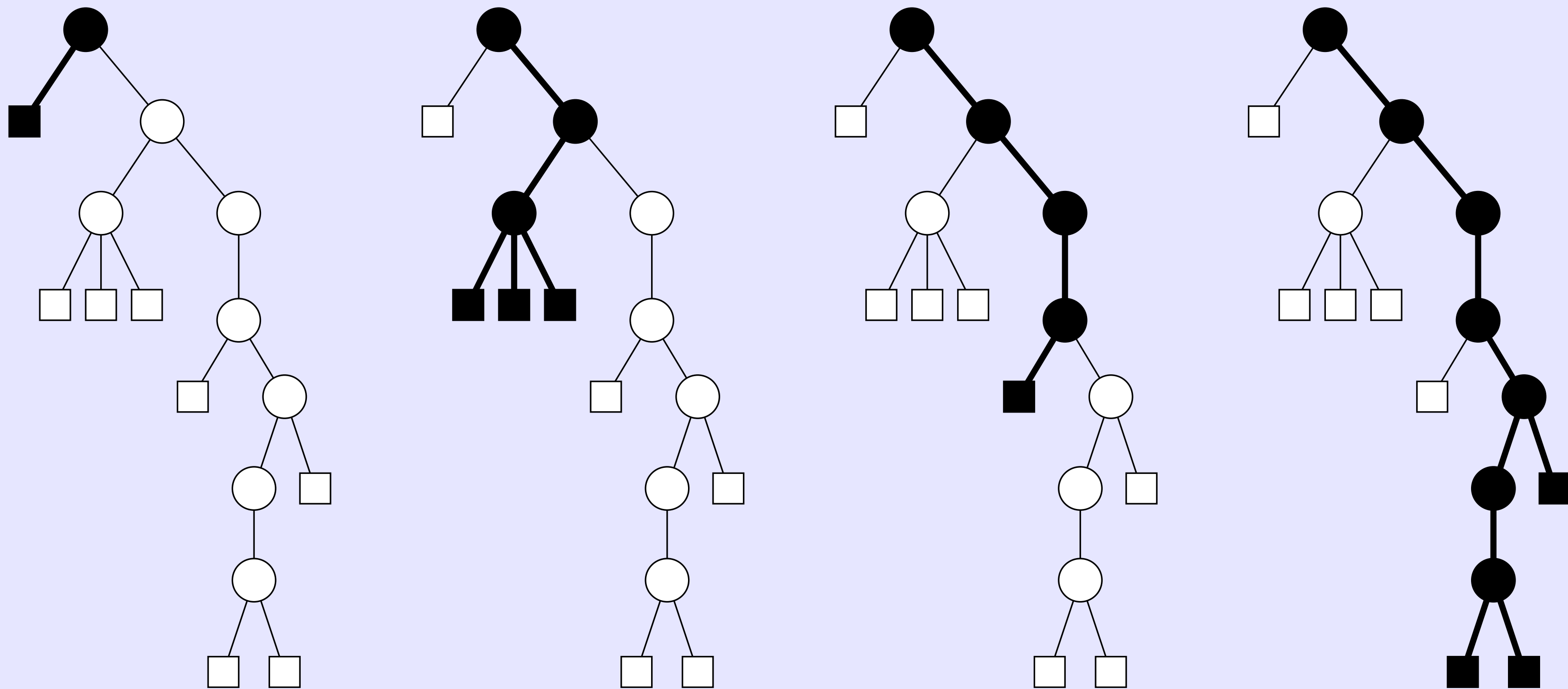


- A prefix of length p of a path in a giraffe tree using a BFS layout can be traversed in $O(p/B)$ I/Os

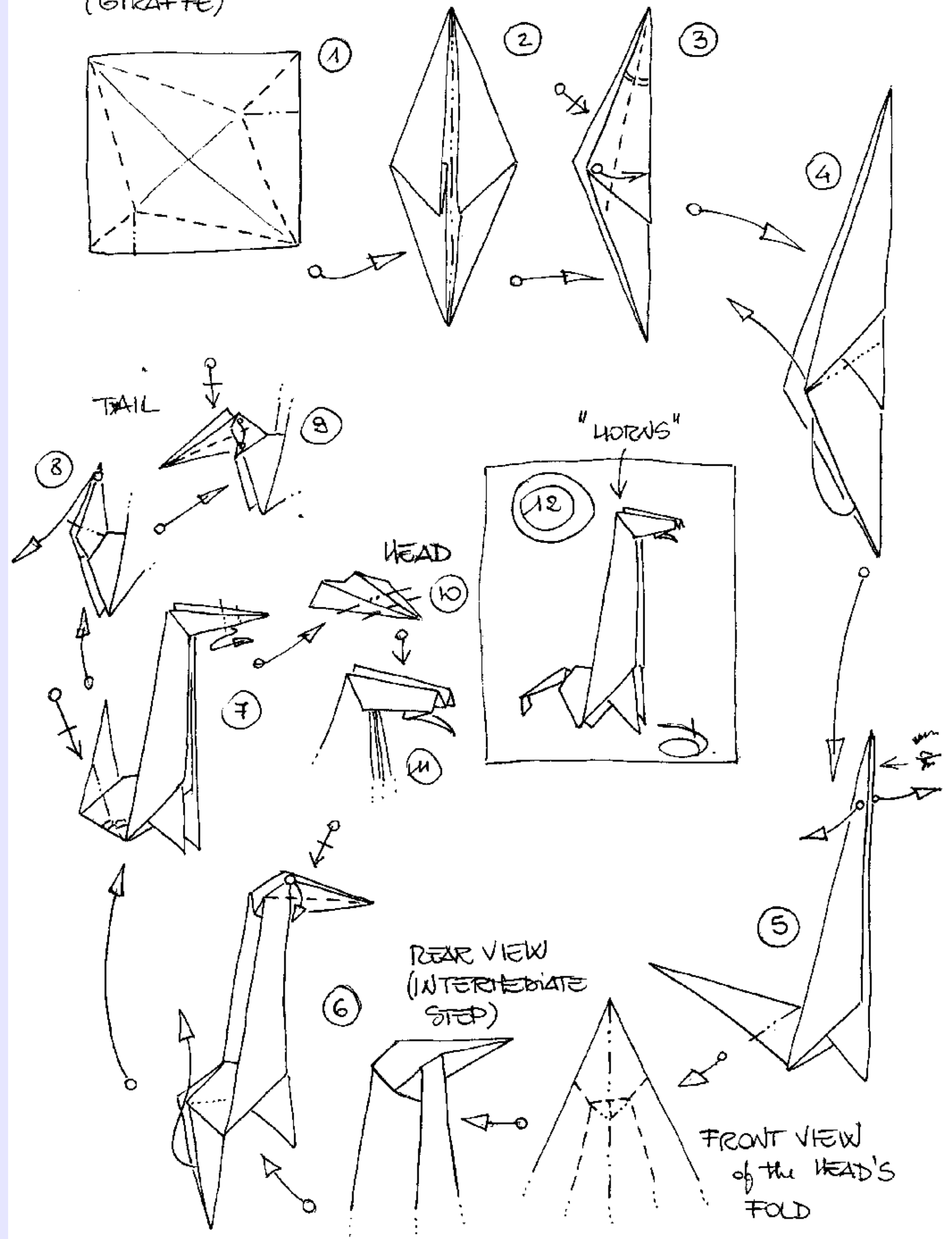
Giraffe Cover of a Tree



Giraffe Cover of a Tree



- Uses space $O(N)$ and can be constructed greedily from left-to-right using $O(N/B)$ I/Os by an Euler traversal of T
- BFS layout of each giraffe
- A prefix of length p of a path in a known giraffe can be traversed in $O(p/B)$ I/Os


GIRAFFA (1995) diagrammed 1999.
(GIRAFFE)
TAIL
"HORNS"
HEAD
REAR VIEW
(INTERMEDIATE
STEP)
FRONT VIEW
of the HEAD'S
FOLD

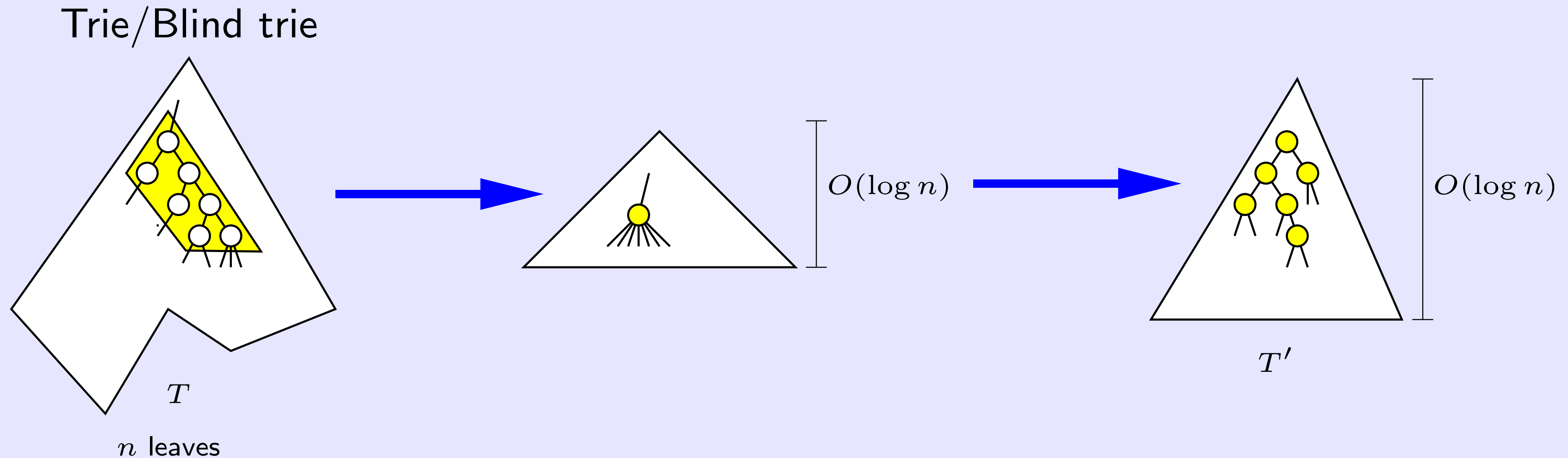
Summary so far...

String dictionary search Suffix tree search Trie search	} reduce to blind trie search

Query : Blind trie search + $O\left(1+\frac{|P|}{B}\right)$ I/Os

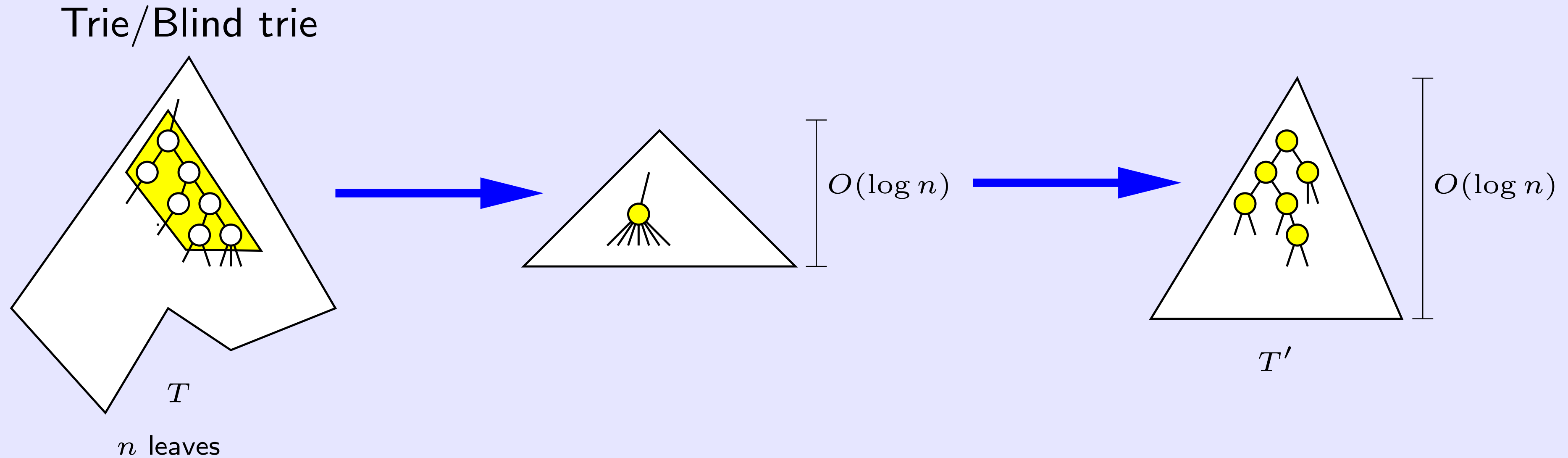
Cache-Oblivious (Blind) Tries

Cache-Oblivious (Blind) Tries



- Partition input trie T into components (generalization of heavy paths)
- T' = collapse components in T into high degree nodes and replace by weight balanced trees
- Apply van Emde Boas layout out to T'

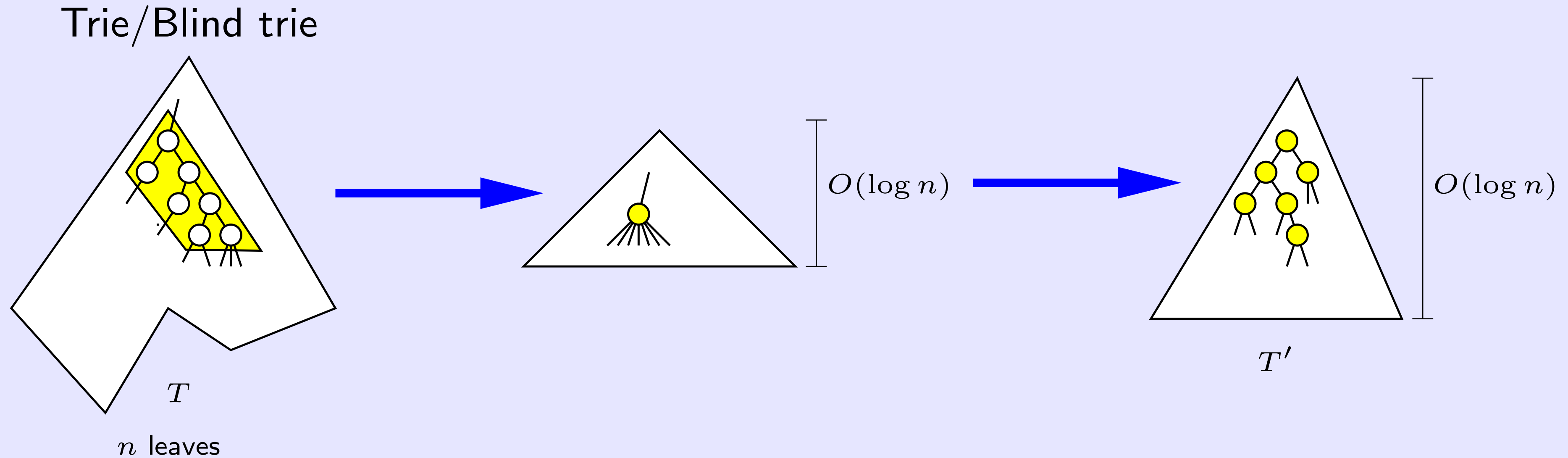
Cache-Oblivious (Blind) Tries



- Partition input trie T into components (generalization of heavy paths)
- T' = collapse components in T into high degree nodes and replace by weight balanced trees
- Apply van Emde Boas layout out to T'

Search: $O(\log_B n)$ I/O

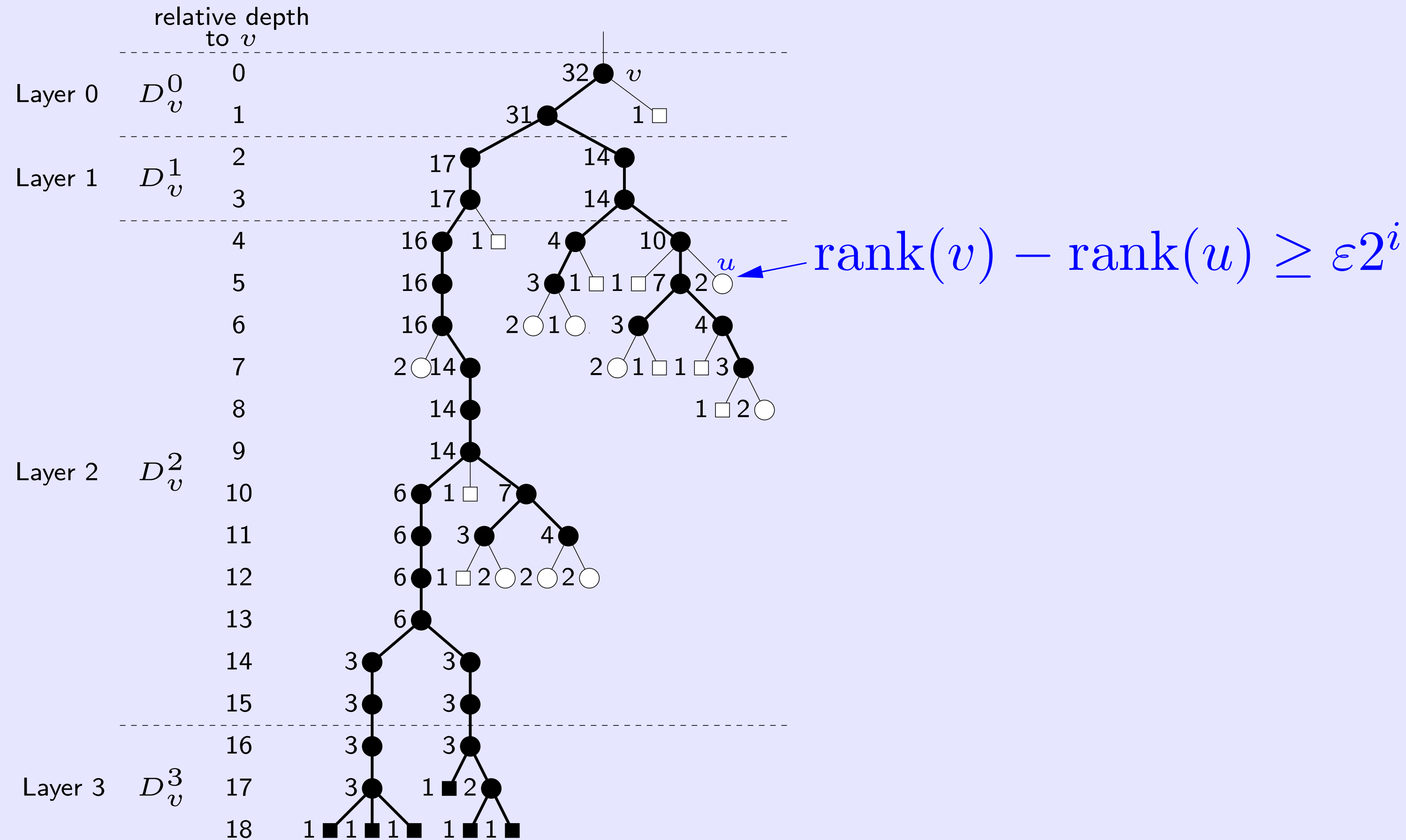
Cache-Oblivious (Blind) Tries



- Partition input trie T into components (generalization of heavy paths)
- T' = collapse components in T into high degree nodes and replace by weight balanced trees
- Apply van Emde Boas layout out to T'

Search: $O(\log_B n)$ I/O — ignoring searching inside components

Decomposition into Components

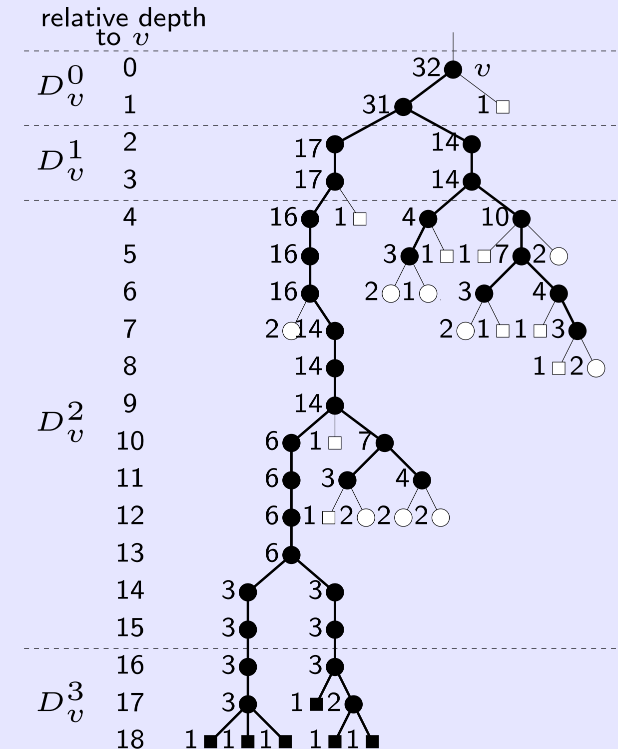


$$D_v^0 = \{u \in T_v \mid \mathrm{rank}(u) = \mathrm{rank}(v) \wedge \mathrm{depth}(u) - \mathrm{depth}(v) < 2^{2^0}\}$$

$$D_v^i = \{u \in T_v \mid \mathrm{rank}(v) - \mathrm{rank}(u) < \varepsilon 2^i \wedge 2^{2^{i-1}} \leq \mathrm{depth}(u) - \mathrm{depth}(v) < 2^{2^i}\}$$

Storing and Searching Components

- Store each layer D_v^i separately
- Make a giraf-decompostition of D_v^i
- For D_v^i have a blind trie
  of size $O(2^{\varepsilon 2^i})$ (using BFS layout)
  to select the right giraffe-tree
- Search: D_v^i search the blind trie +
  search in one giraffe-tree
- Distribute $D_v^0, D_v^1, D_v^2, \ldots$
  in the van Emde Boas layout of T'



- Analysis:
  - Search in blind trie for D_v^{i+1} dominated by the matched characters in D_v^i
  - Space in van Emde Boas layout for a subtree of size k becomes $O(k^3)$

Cache-Oblivious Tries

There exists a cache-oblivious trie supporting prefix queries in

$$O(\log_B |n| + |P|/B) \text{ I/Os},$$

where P is the query string, and n is the number of leaves in the trie.

It can be constructed in $O(\mathrm{Sort}(N))$ time, where N is the total number of characters in the input.

The space required is $O(N)$.

The structure assumes $M \geq B^{2+\delta}$.

Conclusion

- A string dictionary (trie data structure) was presented that supports queries in $O(\log_B n + |P|/B)$ I/Os. The data structure uses $O(N)$ space and can be constructed using $O(\mathrm{Sort}(N))$ I/Os.
- Lookahead in the query string is crucial (both cache-aware and cache-oblivious)
- A giraffe cover is a simple construction allowing topdown path traversals in a tree using $O(|P|/B)$ I/Os

Open problems

- Prove a lower bound trade-off between the number of I/Os required for a query and the lookahead used
- Implementation: compare with string B-trees, tries, ternary trees, different trie layouts, ...

Thank you —Lunch !